3-21-2019

Fuzzy Inference Systems for Risk Appraisal in Military Operational Planning

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FUZZY INFERENCE SYSTEMS FOR RISK APPRAISAL IN MILITARY OPERATIONAL PLANNING

THESIS

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AFIT-ENS-MS-19-M-141

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THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

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March 2019

DISTRIBUTION STATEMENT A.
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

AFIT-ENS-MS-19-M-141

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Abstract

Advances in computing and mathematical techniques have given rise to increasingly complex models employed in the management of risk across numerous disciplines. While current military doctrine embraces sound practices for identifying, communicating, and mitigating risk, the complex nature of modern operational environments prevents the enumeration of risk factors and consequences necessary to leverage anything beyond rudimentary risk models. Efforts to model military operational risk in quantitative terms are stymied by the interaction of incomplete, inadequate, and unreliable knowledge.

Specifically, it is evident that joint and inter-Service literature on risk are inconsistent, ill-defined, and prescribe imprecise approaches to codifying risk. Notably, the near-ubiquitous use of risk matrices (along with other qualitative methods), are demonstrably problematic at best, and downright harmful at worst, due to misunderstanding and misapplication of their quantitative implications. The use of fuzzy set theory is proposed to overcome the pervasive ambiguity of risk modeling encountered by today's operational planners. Fuzzy logic is adept at addressing the problems caused by imperfect and imprecise knowledge, entangled causal relationships, and the linguistic input of expert opinion. To this end, a fuzzy inference system is constructed for the purpose of risk appraisal in military operational planning.

Acknowledgments

I am obligated by conscience and sincerity to submit my foremost gratitude to my wife; it is upon her saintly patience that any good to come of this thesis exclusively rests. I would also like to express my indebtedness to the wit and wisdom of my faculty advisor, Dr. Richard F. Deckro, and to my reader, Lt Col Matthew J. Robbins, to whom this effort owes much.

Curtis B. Nelson

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FUZZY INFERENCE SYSTEMS FOR RISK APPRAISAL

IN MILITARY OPERATIONAL PLANNING

I. Introduction

> *"The art of war deals with living and with moral forces. Consequently, … it must always leave a margin for uncertainty, in the greatest things as much as in the smallest. […] Mathematical factors never find a firm basis in military calculations. From the very start there is an interplay of possibilities, probabilities, good luck and bad that weaves its way throughout the length and breadth of the tapestry."*
>
> - Carl von Clausewitz
> *On War (1832, 86)*

1.1 General Issue

Clausewitz's seminal work on the theory of war certainly does not advocate for the wholesale abandonment of empirical method in military planning and decision-making. Rather, he suggests that while it is "quite clear how greatly the *objective nature* of war makes it a matter of assessing probabilities," it is precisely the confluence of this uncertainty with the element of chance, derived predominantly from the human element, that defines the *subjective nature* of war and that makes its conduct, relative to any other human activity, a gamble (Clausewitz, 85). It is prudent that military strategists and policymakers employ various methods to quantify the odds, risks, and opportunities of this deadly gamble. Nevertheless, the reader is cautioned that quantitative analysis should not direct the dogmatic application of prescriptive formulation; the complexity and constant change inherent to war prohibit this. Instead, rigorously applied principles are "indispensable to… the theory of war that leads to positive doctrine; for in these doctrines the truth can express itself only in such compressed forms" (Clausewitz, 152).

The violence and politics that Clausewitz observed on Napoleonic battlefields is no less central to modern conflict, but technology and globalization have since facilitated the natural extension of warfare into the domains of cyberspace, extra-atmospheric space, and perception space; each of these domains possessing unique and increasingly diverse means of waging combat. This nonlinear and multi-dimensional battlespace obfuscates the *coup d'oeil*, or acuity for innate truth, of even the most gifted commanders, who were once advised to "familiarize himself only with those activities that empty themselves into the great ocean of war" (Clausewitz, 144). In keeping with this analogy, the rivers discharging into today's ocean of warfare are vast in number, each fed by a multitude of tributaries riddled with unique hazards perceptible only to experienced helmsmen. Correspondingly, commanders are progressively dependent on the informed analysis and communication of risk by expert subordinate staff who must fine-tune their senses to pierce the veil of Clausewitzian fog that obscures their specific risk domains. Opportunities to further inform military planning processes also exists in the prevalence of data and the promise of machine learning, driven by the increasing digitalization of maneuver forces and command and control systems.

1.2 Problem Statement

Advances in computing and mathematical techniques have given rise to increasingly complex models employed in the management of risk across numerous disciplines. While current military doctrine embraces sound practices for identifying, communicating, and mitigating risk, the nature of modern operational environments frustrates the enumeration of risk factors and consequences necessary to leverage anything beyond rudimentary risk models. Efforts to model military operational risk in quantitative terms are stymied by the interaction of incomplete, inadequate, and unreliable knowledge. Recognizing the limitations of strict mathematical formulation in evaluating risk, current Department of Defense (DoD) and Service literature are necessarily vague in advocating for the use of numerical techniques, instead insisting on the primacy of qualitative assessments utilizing a common lexicon of linguistic categorization. Inflexibly dependent on the persistent input of expert opinion, such methodologies are inherently plodding and unresponsive to reformulation, are vulnerable to inconsistency in subjective judgment, and disallow the comprehensive assessment of risk under meaningful singleton values for the purpose of course of action comparison.

1.3 Research Objective

It is the objective of this thesis to begin the development of a viable method for the quantitative assessment of military operational risk in joint planning.

1.4 Investigative Questions

Oriented on the research objective, four investigative questions (IQ) are employed to structure the direction and content of the research.

IQ1. How is operational risk addressed in current joint and Service literature?

IQ2. What challenges are presented by the current doctrinal means of quantitative risk evaluation?

IQ3. What are the characteristics of fuzzy logic that suggest its ability to reconcile quantitative risk evaluation with its inherent challenges?

IQ4. Is the proposed model, a fuzzy inference system, suitable for the quantification of risk within the current military planning and risk frameworks?

Each IQ is independently examined in the subsequent chapters. Chapter II addresses the first three in sequence: IQ1 in Section 2.2, IQ2 in Section 2.3, and IQ3 in Section 2.4. Chapter III illustrates the model's development and relationship to current planning practices. Chapter IV presents the practical results of an example scenario. Taken together, these latter two chapters support the analysis of IQ4. Chapter V formally presents the summary answers to all of the thesis' investigative questions.

1.5 Methodology

The use of fuzzy set theory is proposed to overcome the pervasive ambiguity of risk modeling encountered by today's operational planners. Fuzzy logic is adept at addressing the problems caused by imperfect and imprecise knowledge, entangled causal relationships, and the linguistic input of expert opinion (Shang, 3). Specifically, a *fuzzy inference system* is introduced that capitalizes on the current construct of the Joint Planning Process' (JPP) information requirements to inform model construction as a natural byproduct of planning and that subordinates itself as the quantitative engine of the Joint Risk Analysis Methodology (JRAM). Fuzzy inference systems encode functional expertise through a logical rule base that manipulates linguistic variables and ambiguous categorizations, ultimately producing an actionable and discrete output. As the JPP and JRAM are largely mimicked by the individual Service doctrines, the proposed methodology is generalized for use across the DoD and for operational risk assessments at the strategic, operational, or tactical level.

1.6 Assumptions and Limitations

By definition, fuzzy inference systems emulate human deductive reasoning through the establishment of a logical rule base encoded from expert opinion (Kosko, 25). Furthermore, an inference model's linguistic variables (including the shapes and quantities of their corresponding membership functions) are reliant on an extensive knowledge base predicated on available experience data or, again, subject matter expertise. Therefore, the ability to formulate a fuzzy inference system is heavily dependent on the existence and input of expert opinion. This dictates the assumption that the risk analyst have unconstrained access to necessary expertise during a model's construction.

Secondly, a primary advantage in using a fuzzy approach lies in its ability to deal with imprecision and ambiguity. In the context of military operational risk, other deterministic or probabilistic methods may involve arrogant prescriptions resulting in overly precise, but less accurate, results. Alternatively, fuzzy systems exchange precision for accuracy; they do not guarantee optimality even under conditions of omniscience. Rather, the degree of constituent set fuzziness correlates with the model's range of precision; it is assumed that this level of precision is sufficient and that the solution thus derived is acceptably accurate. This effect may be more readily recognized when output risk levels are defined in concrete terms like cost or casualty rate, as opposed to a generic 'risk level.'

With regard to this preceding concern, it is certainly possible for fuzzy output variables to be defined in explicit and tactile terms. However, this thesis assumes that the model is predominantly utilized in comparative processes (for instance, course of action comparison in operational planning). The meaning of a risk value must be assessed relative to other risk values generated by the same model; comparing results from dissimilar models may encourage inconsistent risk decisions. This is not to suggest a paramount rigidity in application; fuzzy inference systems are flexible and easily accept modifications based on the emergence of new data or change to expert opinion. Indeed, better informed models are likely to have more accurate resolution. However, given a change to a model, all considered alternatives would require reassessment.

While trivial instances of fuzzy inference systems may be evaluated manually, the volume of calculation necessitates the use of computer-based models in any practical scenario. A number of commercial tools are available for building and evaluating fuzzy control and fuzzy inference systems. While not available to most military staffs at present, it is assumed that the risk analyst has access to software or a programming language that facilitates the implementation of fuzzy models. This thesis employs MATLAB’s Fuzzy Logic Toolbox in the construction and analysis of the example scenario presented in Chapter IV.

1.7 Preview

This research is structured in four succeeding chapters. Chapter II, Literature Review, examines three dominant themes. First, military operational risk is defined and explored in a general sense, but also from the joint and Service perspectives. In answering IQ1, this Section illuminates several doctrinal deficiencies; conspicuously, the combined literature is inconsistent in the representation of risk and reluctant to endorse quantitative practices beyond vague equivocations. Secondly, and addressing IQ2, several concerns with risk matrices are addressed, both as a conceptual framework for understanding risk and as the principal means of conveyance and visualization in military parlance. Third, the chapter presents an elementary but thorough introduction to fuzzy set theory and the mathematical principles necessary for the model's execution. This portrayal of fuzzy logic is expectedly indicative of its utility as a method for dealing with insufficient and imprecise data, a critical aspect of the challenge posed by IQ3. Chapter III, Methodology, presents a detailed description of the proposed model in two phases. The first phase, knowledge elicitation, informs the model's construction as a parallel procedure to the JPP. The second phase, execution of the inference engine, applies the mathematical principles introduced in Chapter II to the constructed model to obtain a quantitative output and visual representation. Chapter IV, Analysis, demonstrates use of the model through the fictional scenario of a tactical rotary wing mission. This chapter illustrates a practical use of the methodology and suggests at its suitability, in response to IQ4, as a model for risk appraisal and decision-making. Finally, Chapter V, Conclusion and Recommendations, summarizes the research and suggests its significance, its potential for use, and areas for future research.

II. Literature Review

"Even in reasoning upon some subjects, it is a mistake to aim at an unattainable precision. It is better to be vaguely right then exactly wrong."[1]

- Carveth Read
Logic, Deductive and Inductive (1898, 351)

2.1 Chapter Overview

The purpose of the chapter is threefold. Sections 2.2 through 2.2.2 examine some historical notions of risk and attempts at quantification, but primarily serve as a brief survey of the current doctrine of the US Department of Defense and its subordinate Services as it pertains to risk assessment and management. Section 2.3 presents a criticism of commonly practiced qualitative methodologies; several latent complications introduced by the military's prosaic use of risk matrices are discussed. Lastly, Sections 2.4 through 2.4.3 provide a primer on fuzzy logic, its basic set theory, membership functions, and operations on fuzzy sets.

[1] Read, Carveth. (1920). *Logic, Deductive and Inductive*, 4th Ed. London: Simpkin, Marshall, Hamilton, Kent & Co. Ltd. The latter half of this quote is often misattributed to John Maynard Keynes. The famous economist's father, John Neville Keynes, published *Studies and Exercises in Formal Logic* (1884) in attempt to synthesize deductive and inductive reasoning, and is cited prominently by Read.

2.2 Military Operational Risk

Writing in the early part of the nineteenth century, at the culmination of the Napoleonic Wars and prior to the full realization of the Industrial Revolution, Clausewitz chronicled his insight on what was perhaps the most considerable human enterprise of the time, warfare. Although he identified uncertainty as a pillar of war's "paradoxical trinity" (along with primordial violence and its subordination to politics), the concept of risk is not explicitly defined. Clausewitz' discourse on the subject, while exhaustive, can largely be reduced to dependency on individual talent and luck. Contemporaneously, the decline of mercantilism and an emerging free-market economy provided an impetus for the mathematical treatment of the role of risk, as in Adam Smith's *The Wealth of Nations* (1776), which questioned the utility of classical probability theory to decision-making based on amorphous and indeterminate informational constructs (Smith, 1776). Much later, and subsequently motivating Great Depression era studies in macroeconomics, formalized structures for calculating probabilities in the context of risk were introduced in John Maynard Keynes' *A Treatise on Probability* (1921), presenting a degree-of-truth permitting interval approach to probability theory, and Frank Knight's *Risk, Uncertainty and Profit* (1921), which notably distinguished risk and uncertainty relative to whether the associated probability distribution was known.

Wishing to avoid a comprehensive survey of uncertainty and probability, it is sufficient that the desire to understand to the role of ambiguity in business, finance, and economics continued to stimulate the evolution of quantitative risk analysis throughout the twentieth century. Over time, the concepts derived from the science of economic risk were adapted for use in other disciplines; the practice of risk management is virtually ubiquitous in all major endeavors, ranging from healthcare, technology, energy, construction, project management, to defense and finance, among others. Indeed, many of the current academically accepted formulations and definitions in risk management are those proposed by the Basel Committee on Banking Supervision's *International Convergence of Capital Measurement and Capital Standards: A Revised Framework* (2004), which is commonly referred to as the "*Basel II Accord*." While predominantly concerned with monetary policy and the establishment of an infrastructure for managing for capital adequacy relative to risk exposure in international banking, *Basel II* pertinently segregates operational risk from the other risk categories of credit risk and market risk. Specifically, it defines Operational Risk (OR) as "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events" (BCBS, 2004).

While the *Basel II* definition of OR is equally useful in conceptualizing the risks inherent to organizations engaged in armed conflict, the exact approaches used to calculate the associated capital requirements have little utility in this regard. Nevertheless, the shared sources of risk definition and management practices are reflected in the close taxonomical resemblance of military risk with its economically-oriented counterparts. Notably, the Chairman of the Joint Chiefs of Staff Manual (CJCSM) on *Joint Risk Analysis* (2016) cites a white paper published by the International Risk Governance Council (IRGC), *Risk Governance: Towards and Integrative Approach*, as foundational to the Department of Defense's top-level literature on risk (Renn, 2006). Similarly used to shape the DoD's risk framework is the International Organization for Standardization's (ISO) publication, *Risk*

Management – Principles and Guidelines (2009). By the same token, the very impetus for this thesis' methodology was in part stimulated by a series of joint studies sponsored by the Casualty Actuarial Society, the Canadian Institute of Actuaries, and the Society of Actuaries that explore the applicability of several fuzzy logic practices for risk management (Shang & Hossen, 2013; Shapiro & Koissi, 2015). A related effort, but singularly oriented on the specific technique of fuzzy inference, was also made on behalf of Colombia's central bank (Reveiz & León, 2009).

International project management, as one of the few fields that approaches the broad risk exposure experienced in military campaigns, is also potentially informative to the assessment of military risk. The prospect of achieving commercial success in underdeveloped but high-demand markets has motivated individual risk practitioners to scrutinize the complexities of international construction ventures and the accompanying difficulties in identifying critical risk contributors as studied by Kerur & Marshall (2012) and Li (2009), while more extensive studies have been funded by the industry at large (Gibson & Walewski, 2004). Certainly, it is evident that many lucrative speculations are fraught with the challenges posed by diverse geographic environments, poor infrastructure, and corrupt or ineffective governance, to name a few. Future models for military operational risk may increasingly parallel the risk assessment structures present in this activity. Closely related to this concern is the development of a suitable catalogue for classification of global and country-specific risk factors. Many diverse efforts have been made to derive the key risk indicators for loss potential in uncertain environments (Anderson, Hager, & Vormeland, 2016), to model nation-state instability via multivariate methods (Shearer & Marvin, 2012), and finally to model the risk drivers of construction cost performance utilizing fuzzy decision frameworks (Baloi & Price, 2003).

2.2.1 Risk in the Department of Defense

The capstone of all risk literature in the DoD is *Joint Risk Analysis* (CJCSM 3105.01, 2016), which has the stated purpose of establishing a "Joint Risk Analysis Methodology [and introducing] a common risk lexicon to promote consistency across the Joint Force." While specifically oriented on supporting risk management practices at the Joint Chiefs of Staff level and, in particular, for use in the "Strategic Planning Construct," it remains the authoritative reference on risk for the Services, Combatant Commands, joint activities, and certain defense agencies and is applicable "across the entire spectrum of their responsibilities." The JRAM is designed to standardize a framework of risk-decision processes and taxonomy that institutes best practices to evaluate, manage, and communicate comprehensive risk. Depicted in Figure 1, the Joint Risk Framework consists of three components (*Risk Appraisal, Risk Management*, and *Risk Communication*) and four subordinate activities (*Problem Framing*, *Risk Assessment*, *Risk Judgment*, and *Risk Management*). The illustration shows the cyclical conduct of the four activities, each posing a distinct question that respectively yields, for a given problem, well-defined and context-specific risk conventions, the identification and weighting of threats or hazards, a risk profile and quantitative evaluation, and concludes with mitigation actions and risk decisions (JRA, B-1).

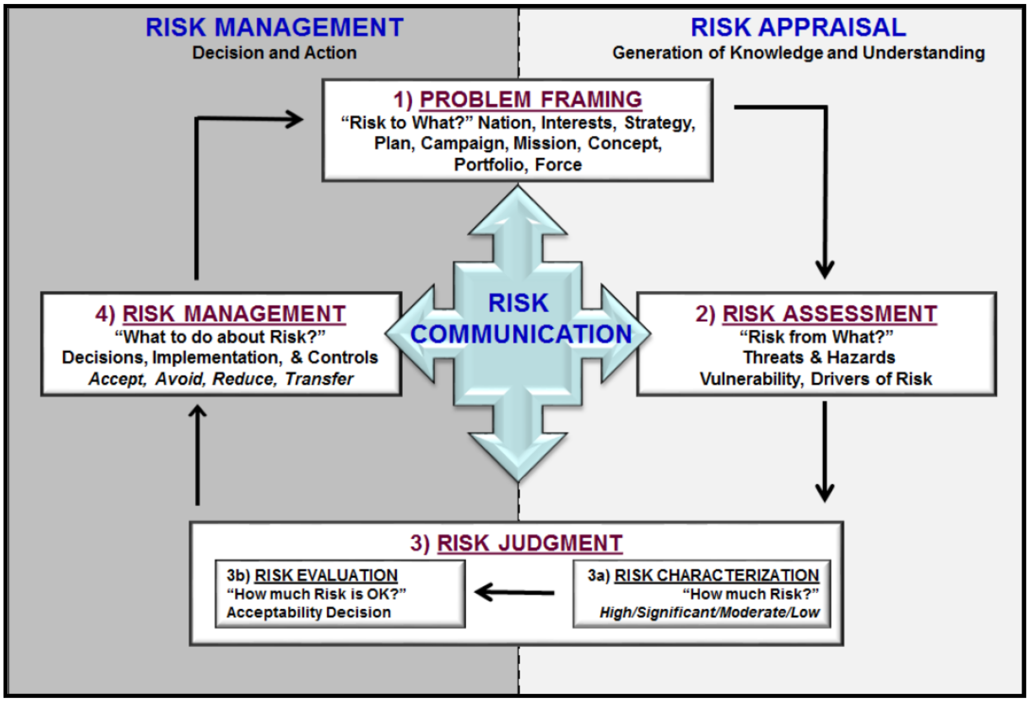


Figure 1. The Joint Risk Framework.
Source: Joint Risk Analysis, B-2.

Risk, under the JRAM, is defined as the "probability and consequence of an event causing harm to something valued" (JRA, B-1). In this context, *Problem Framing* retains the traditionally accepted risk conventions of "likelihood (probability) of event occurrence" and "severity (consequence) of harm caused." In the JRA manual, probability and consequence are both divided into the four categories reflected in Table 1, while suggesting only several pages later that a more appropriate degree of categorization be five (JRA, B-7). Nevertheless, the document is careful not to be overly prescriptive; it recognizes that probability and consequence must be tailored to the specific risk scenario. For instance, an assessment of ~20% chance of occurrence in certain risks, like that of an aircraft shootdown, dubiously warrant a 'Highly Unlikely' linguistic appraisal; such a high probability is perhaps more contextually appropriate as 'Very Likely.' Similarly, the measure of acceptable damage to critical infrastructure may be more restrictive than to that of more ordinary materials; evaluation of the two against the same scale or criteria, even when sharing a common metric (say, in monetary terms), is likely inappropriate. The natural language descriptions of risks, probabilities, and consequences often convey more information than ascription of a single numeric value. Conversely, the imprecision of language is simultaneously problematic even considering context; for instance, it is a rhetorical exercise to ask how 'Major' and 'Moderate' consequences are distinguished. While the JRAM is cognizant of the susceptibility to fallacy, it offers only acknowledgement in consolation, not resolution.

Table 1. JRA Probability, Consequence, and Risk Levels.
Source: Author's elaboration from Joint Risk Analysis, B-2, B-3, B-5, C-10.

Probability (P)	**Consequence** (C)	**Risk Level** $Risk = f(P, C)$	**Military Risk Description**
Very Likely (~81 – 100%)	Extreme harm to something of value	High – Extreme expected impact	• Mission Failure, Objectives Unachievable • No Sourcing Solutions Exist for Critical Requirements
Probable (~51 – 80%)	Major harm to something of value	Significant – Unacceptable expected impact	• Objectives Partially Achieved (consider time, priority) • Shortfalls Exist for Critical Requirements
Improbable (~21 – 50%)	Moderate harm to something of value	Moderate – Maximum acceptable expected impact	• Objectives Partially Achieved (consider time, priority) • Worldwide Sourcing Solutions Exist for Most Requirements
Highly Unlikely (~0 – 20%)	Minor harm to something of value	Low – Little or no expected impact	• Mission Success, Objectives Achievable • Joint Force Fully Sustained and Requirements Sourced

The JRAM's *Risk Assessment* activity attempts to establish the causal linkages between the sources of risk, their drivers, and the occurrence of the harmful event. A distinction is first made between threat and hazard sources; the former is an entity that actively intends harm, the latter is comprised of the passive potential of some condition to result in harm. Risk drivers are defined to be any factor that alters the risk expectation through manipulation of probability or consequence. The examined considerations may include the object, event, or idea of interest's vulnerability to harm, its resilience from harm, its importance, accessibility or exposure to the threat or hazard, as well as analysis of the higher-order effects of its damage or loss. The *Risk Judgment* activity then calculates the individual threat and hazard risk level as a function of that threat's estimated probability and consequence, $Risk = f(P, C)$. The JRAM does not, however, go on to define this functional relationship further, notably refraining from their commonly recognized multiplicative interaction. As a result, the visual representation of risk level is indicated on an ostensibly ambiguous, but intentionally continuous, contour graph; again, this is an apparent departure from the near-ubiquitous use of the 'risk matrix' in inter-Service literature. In this sense, Figure 2's "Risk Contour Graph" is used an aid for subject matter experts and decision-makers in the subjective assignment of risk level utilizing any previously calculated values as bounds or approximations.

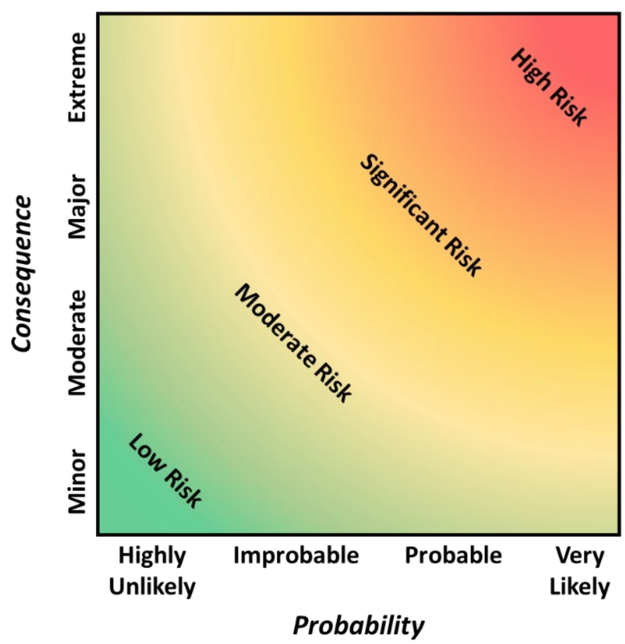


Figure 2. Risk Contour Graph.
Source: Author's elaboration from Joint Risk Analysis, B-5.

The actual risk judgment is one of acceptability; an intolerable risk level either warrants additional constraints or the application of limited resources to mitigate the threat's probability or consequence. *Risk Management* addresses this concern through any of four techniques:

- Acceptance; an informed decision to act without mitigation,
- Avoidance; the risky activity is abandoned altogether,
- Reduction; mitigation strategies are employed to lower the risk,
- Risk Transfer; shifting where, when, and to whom the risk is incurred.

Regarding *Risk Communication*, the JRA importantly acknowledges that any methodology must be conjoined with sound military judgment in addressing operational risk; while the JRAM attempts to establish a common system that facilitates collective risk communication, alternative frameworks should be utilized when situationally appropriate. Regardless of the model employed, effective communication is paramount; the language of risk should be easily understood across domains and organizations. Without specificity or context, the statement that a given scenario is "High" risk, for example, may breed confusion and ultimately result in a suboptimal risk decision. Instead, productive risk dialogue is contingent on tangible articulation in terms of "actual costs, options, impacts, and end-states" (JRA, A-4).

The JRA also distinguishes *Strategic Risk*, which is focused on impact to national interests, from *Military Risk*, which is concerned with threat to the Joint Force (Risk-to-Force) and the ability to accomplish military objectives (Risk-to-Mission). While the force management and institutional risks of Risk-to-Force are beyond the scope of this thesis, Risk-to-Mission contains the subsets of Operational Risk (OR) and Future Challenges Risk. Of interest to this paper, OR may be defined as a function of the probability and consequence of the current force's failure to accomplish "current, planned, and contingency operations in the near-term (0-2 years) … within acceptable human, material, and financial costs" (JRA, C-8). Formally, Operational Risk is assessed in light of the military objectives called for under the current National Military Strategy (NMS); the principal sources for evaluation of OR are Campaign Plans, Crisis Response Execution Orders (EXORDs), Guidance for Employment of the Force (GEF) objectives, and Global Force Management (GFM) directives. Again, while the JRAM is specifically constructed for risk evaluation at a strategic level, and Operational Risk correspondingly defined, it is not incorrect to consider OR, in broad terms, as assessing the ability of any echelon, at any level of warfare, to accomplish a currently assigned military objective at acceptable cost. This paper subscribes to such a definition; the proposed methodology for evaluating military operational risk is, like the JRAM, a general framework structured within the Joint Planning Process but applicable across the entire spectrum of joint activities. It is notable that the military definition of operational risk considers only the ability to meet operational objectives, not the endogenous force management and institutional risks that ostensibly correspond to concerns of organizational effectiveness in the definition's civil counterpart.

Another important distinction must also be made between military operational risk, environmental safety and occupational health (ESOH) risk, and technical risk, the latter of which is employed here as an umbrella term to describe the programmatic risk encountered in defense acquisition, lifecycle management, and information technologies. While each of these diverse activities is subject to distinct regulatory guidance, it is necessary to highlight a technical risk document whose contents regularly surface not only in DoD operational risk literature, but are also recognized in a variety of governmental, commercial, and international publications. The *Department of Defense Standard Practice: System Safety* (MIL-STD-882E, 2012) institutes a Systems Engineering approach for the risk management of systems, equipment, and infrastructure from inception to grave. Governing risks in a more controlled environment, *System Safety* provides numerical examples of probability and severity criteria that are considerably less vague than those in the JRAM. The probabilities listed in Table 2, while appropriate for engineering applications, are defined over so tight a range as to have no practical utility in an operational sense. The same is potentially true of the dollar value or descriptive categorizations.

Table 2. System Safety Probability, Severity, and Risk Levels.
Source: Author's elaboration from System Safety (MIL-STD-882E, 11, 12, 91).

Probability (P)	**Severity** (S)	**Risk Level** $Risk = P \times S$	**Risk Description**
Frequent ($P \geq 10^{-1}$)	Catastrophic ($S \geq \$10M$)	High	• Death, Perm. Total Disability; Irreversible Signif. Environ. Impact • Continuously Experienced
Probable ($10^{-1} > P \geq 10^{-2}$)	Critical ($\$10M > S \geq \$1M$)	Serious	• Permanent Partial Disability, Injury, or Illness; Signif. Environ. Impact • Occurs Frequently
Occasional ($10^{-2} > P \geq 10^{-3}$)	Marginal ($\$1M > S \geq \$100K$)	Medium	• Injury or Occupational Illness; Reversible Mod. Environ. Impact • Will Occur Several Times
Remote ($10^{-3} > P \geq 10^{-6}$)	Negligible ($\$100K > S$)	Low	• Injury or Occupational Illness; Minimal Environmental Impact • Unlikely, but can reasonably be expected to occur.
Improbable ($10^{-6} > P$)			• Possible, but so unlikely it can be assumed occurrence may not be experienced.

System Safety is also, however, cautious in suggesting any fixed figures for categorical assignment and, while giving preference to quantitative data, ultimately demands compliance with the qualitative descriptions in the absence of frequency or rate data (MIL-STD-882E, 12). Several additional observations may be made from Table 2 that distinguish it from the JRAM. First, it prescribes six probability categories (the category for ‘Eliminated’ is not shown) and only four for Severity and Risk Level. Second, risk is an expectation; it is the product of probability and severity. Lastly, the actual language used in defining the categories is different and those linguistic categories do not retain the same meaning from those in the JRAM. This is particularly true as the transformation from probability and severity to risk level is defined by a discretely categorized risk matrix (with compulsory compliance), as opposed to the continuous categorization of the JRAM’s risk contour graph. Without understanding these important distinctions, the errant application of this technical risk document by maneuver forces in operational planning is both faulty and dangerous.

2.2.2 Risk in the Uniformed Services

Perhaps tellingly, the conceptual interpretation of risk within the DoD's component Uniformed Service literature is presented in a manner more closely resembling that of *System Safety* than of *Joint Risk Analysis*. The Service policies respecting operational risk are contained in Army Techniques Publication 5-19: *Risk Management* (2014), Marine Corps Order 3500.27C: *Risk Management* (2014), Chief of Naval Operations Instruction 3500.39C: *Operations Risk Management* (2010), and Air Force Instruction 90-802: *Risk Management* (2017). Largely homogenous in content, these documents borrow from each other extensively to the point of using the exact language of several key doctrinal features. The literature almost[2] universally acknowledges four foundational risk management (RM) principles:

- Integrate RM into all Phases of Missions and Operations,
- Make Risk Decisions at the Appropriate Level,
- Accept no Unnecessary Risk,
- Apply RM Cyclically and Continuously.

There also exists a generally accepted formulation for the actual systematic procedure of risk management as a cyclical and continuous five-step process, fundamentally corresponding to that of the JRAM:

- Identify the Hazards,
- Assess the Hazards,
- Develop Controls and Make Risk Decisions,

[2] The Navy consolidates the "Integrate…" and "Apply…" principles into a single bullet and adds the additional principle of "Accept Risk when Benefits outweigh the Cost" (OPNAVINST 3500.39C, 2).

- Implement Controls,
- Supervise and Evaluate.

While these similarities ostensibly suggest consistency in the body of Service literature, while still disparate from the Joint publication, there are notable inconsistencies in the precise taxonomies and methodological frameworks within the first two steps: Identification and Assessment of hazards. Together, these two elements of RM are considered semantically equivalent to the component of *Risk Appraisal* within the context of the JRAM; again, the JRAM's articulation of risk level is made through use of the contour graph.

Risk Assessment Matrix			**PROBABILITY**			
			Frequency of Occurrence Over Time			
			A Likely	B Probable	C May	D Unlikely
SEVERITY Effect of Hazard	I	Loss of Mission Capability, Unit Readiness or Asset; Death	**1**	**1**	**2**	**3**
	II	Significantly Degraded Mission Capability or Unit Readiness; Severe Injury or Damage	**1**	**2**	**3**	**4**
	III	Degraded Mission Capability or Unit Readiness; Minor Injury or Damage	**2**	**3**	**4**	**5**
	IV	Little or No Impact to Mission Capability or Unit Readiness; Minimal Injury or Damage	**3**	**4**	**5**	**5**
Risk Assessment Codes						
1 – Critical	**2** – Serious	**3** – Moderate	**4** – Minor	**5** – Negligible		

Figure 3. Navy Risk Assessment Matrix.
Source: ORM (OPNAVINST 3500.39C), 10.

Herein lies an important distinction; not only do the various Service frameworks employ risk matrices as the primary instrument for the contextualization and translation of hazards to resultant risk levels (as opposed to use of the risk contour), but the Service representations of the risk matrix are dissimilar. For example, Figure 3 depicts the "Basic

Risk Assessment Matrix" as described by the Navy. Meanwhile, the standard risk matrices suggested by the other Services may use contrasting linguistic terms for categorical discrimination, or, even more noticeable, use a different number of probability, severity, and risk level categories altogether. The result is that the topology of the risk matrix is unique to the particular Service; that is, implying that the functional relationship existing between probability and severity is also unique to the particular Service. Consider Figure 4, the Air Force's "Sample Risk Assessment Matrix."

Risk Assessment Matrix				**PROBABILITY**				
				Frequency of Occurrence Over Time				
				A Frequent	B Likely	C Occasional	D Seldom	E Unlikely
SEVERITY	Effect of Hazard	I	**Catastrophic**: Loss of Mission Capability, Unit Readiness or Asset; Death	**EH**	**EH**	**H**	**H**	**M**
		II	**Critical**: Significantly Degraded Mission Capability or Unit Readiness; Severe Injury or Damage	**EH**	**H**	**H**	**M**	**L**
		III	**Moderate**: Degraded Mission Capability or Unit Readiness; Minor Injury or Damage	**H**	**M**	**M**	**L**	**L**
		IV	**Negligible**: Little or No Impact to Mission Capability or Unit Readiness; Min. Injury or Dmg.	**M**	**L**	**L**	**L**	**L**
Risk Assessment Levels **EH** – Extremely High **H** – High **M** – Medium **L** – Low								

Figure 4. Air Force Risk Assessment Matrix.
Source: RM Guidelines and Tools (AFPAM 90-803, 26).

The obvious differences between Figure 3 and Figure 4 are the Air Force's addition of an additional probability category, the removal of a risk assessment level, application of naming conventions to the severity categories, and, most significantly, a wholesale restructuring of the mapping's translational associations from the Navy's symmetrical-about-the-diagonal matrix (which is not necessarily more 'right'). Were isocontour lines to be superimposed over the two figures, it is clear that they would not correspond. In the latter instance, the implication of the isocontour shape is that the Air Force considers an increase in event probability to yield a lower risk than an equivalent increase in event severity. While such a statement may be appropriate in contextually specific scenarios, it is meritless in a general sense, and even particularly dangerous without specification of the numerical ranges under consideration (dangerous in that blind application of the format may result in harmful risk decisions). Furthermore, the language employed between the two risk matrices is not uniform and renders incommensurable any linguistic comparison. For instance, the Navy's highest probability is termed "Likely," while the same term is used for the second highest probability category in the Air Force's matrix. More concerning is the Navy's use of "Critical" as the highest risk level; the Air Force labels the second highest severity category with this word. In the realm of operational risk, where descriptive and qualitative measures are favored for their ability to deal in imprecision, it is evident that the imprecision of language has the potential to convolute inter-Service risk communication.

Finally, it is important to note that neither of the discussed risk matrices are incontestably prescriptive. The Air Force cautions that the presented risk matrix is merely an example and states that "risk assessment matrices can take different forms and should be designed to fit the organization and/or situation as warranted" (AFPAM 90-803, 26). In a similar vein, the Army's publication concedes that

> *"…while mathematics and analytical tools are helpful, Soldiers always need to apply sound judgment. Technical competency, operational experience, and lessons learned weigh higher than any set of alphanumeric codes."*
>
> \- *Risk Management*
> ATP 5-19 (2014, 1-14)

2.3 The Problem with Risk Matrices

One, if not the singularly foremost, reason for the adoption of risk matrices in the management of risk is in its simplicity of use, a simplicity that enables its non-technical employment while simultaneously giving the appearance of the same mathematical rigor and validity intrinsic to strict quantitative methods. Regrettably, the same simplicity belies a treacherous truth; as a qualitative method, risk matrices enjoy, at best, a tenuous purchase on their mathematical underpinnings. It must first be understood that most risk matrices assume a multiplicative relationship within the severity and consequence doublet; this multiplication is, more properly, the formula for *expected* risk. Mathematical expectation captures two desirable properties that make it a meaningful function in the context of risk. First, it trivializes resultant risk when either of the two contributing factors possess a null value. Second, it is monotonic; multiplication results in strictly non-decreasing risk outcomes over any range of nonnegative real severities and consequences. However, the use of expectation as the sole criterion in risk decision-making invites fallacy; the "operation literally commensurates adverse events of high consequences and low probabilities with… events of low consequences and high probabilities" (Haimes, 230). Of course, in light of Taleb's *Black Swan* (2007), it is understood that decision-makers are more often concerned with catastrophic extremes of the former than the humdrum of the latter.

Risk matrices, as qualitative endeavors, also suffer from an inability to satisfy the assumptions necessary for axiomatic application. Their use of expectation assumes they uphold monotonicity; instead of quantitative values, the qualitative risk rankings are assumed to be both non-decreasing and ordinal. Second, it is assumed that their inferential judgments are sound; quantitatively higher risks should be assigned qualitatively higher risk rankings. However, Cox, Babayev, & Huber (2005) theorize that no "direct qualitative rating system satisfying monotonicity is sound for arbitrary quantitative risk functions," to include the multiplicative case (654). To illustrate this point, consider the basic risk matrix shown in Figure 5 and the three circles (cyan, 1; pink, 2; blue, 3; and brown, 4) representing singleton valued quantitative risk expectations calculated as the product of probability and severity. Suppose that severity is scaled over a broad range and that probability is narrowly defined. In such a scenario, it is possible to identify quantitative values of (blue, 3) that exceed that of (cyan, 1), despite the latter assignment of a qualitative 'High' to the former's 'Medium.'

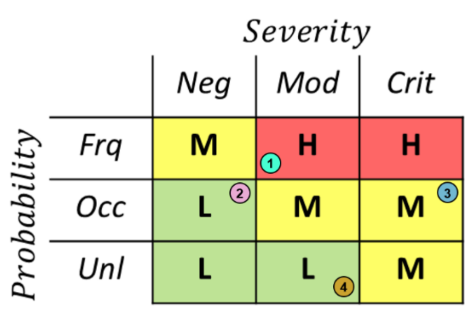


Figure 5. Common Fallacies of Risk Matrices.
Source: Author's elaboration based on the concepts of Cox (2008).

This phenomena of *rank reversal* is demonstrated with the following numerical example. Let probability be defined from 0% to 3%, uniformly distributed, and let severity be defined from \$0 to \$3M, uniformly distributed, then

$$Risk = Probability \times Severity,$$

$$Risk(cyan, 1) = 0.021 \times \$1{,}000{,}001 = \$21{,}000.02, \qquad (1)$$

$$Risk(blue, 3) = 0.019 \times \$2{,}999{,}999 = \$56{,}999.98,$$

and therefore (blue, 3) clearly is higher quantitatively than (cyan, 1), but is assigned a lower qualitative rank. A second phenomena is that of *range compression*, in which the poor resolution provided by the few number of categorical rankings permits assignment of identical qualitative rankings to quantitatively disparate risks. In this manner, Figure 5 fails to distinguish between (pink, 2) and (brown, 4); assuming that (brown, 4)'s probability is asymptotically zero, however, renders (pink, 2)'s quantitative risk orders of magnitude larger, despite having the same ranking. The result is uninformative categorization and inadequate risk management decisions wherein resources cannot be optimally apportioned according to ordinal ranking (Cox [2], 497).

A third and final phenomena is that risk matrices are *error prone* due to discretely delineated categorical boundaries. In short, the true cardinality of quantitative evaluations are obscured for the sake of qualitative ordinality. Regardless of how proximal an expectation is to the nearest boundary, it is assigned exclusive categorization in only one ranking. In Figure 5, assuming that (cyan, 1) is firmly planted in the lower-left corner of its quadrant and that (pink, 2) is in the upper-right corner of its own, then the distance between the two expectations may be infinitesimal so long as it is crosses the boundary. While such an occurrence is problematic in the case of a single category, this example bypasses a category altogether, effectively jumping from 'Low' to 'High.' The failure of the risk matrix to approximate continuous functions is therefore a failure to generalize by induction; uniformly small perturbations potentially produce heterogeneous responses.[3]

[3] This is contradictory to eighteenth-century empiricist David Hume's proposition "that other objects, which are, in appearance, similar, will be attended with similar effects."

The famous pipe. How people reproached me for it! And yet, could you stuff my pipe? No, it's just a representation, is it not? So if I had written on my picture 'This is a pipe', I'd have been lying![4]

- René Magritte
La Trahison des Images (1929)

2.4 Fuzzy Set Theory

Fuzzy logic posits a multivalued set theory, as distinct from classical set theory, that disallows a worldview of unambiguously discrete categorization in favor of one that permits belonging to a particular category as a degree of truth; in other words, suggesting that an object or event may satisfy the conditions of set membership only in part, rather than fully or not at all. Even more concisely, that a set and its complement are not mutually exclusive. Whereas games of chance have definitively distinguishable outcomes and, correspondingly, clear and precise rules for winning and losing, real world problems are inherently noisy, uncertain, and imprecise (certainly there are no such clearly delineated win conditions in modern asymmetrically-waged "grey zone" conflicts). In this regard, "classical probability theory assumes an accuracy and precision of categorization" that is wholly appropriate for predicting the results of coin flips and dice rolls, but potentially inadequate for the modeling of many important problems (Kosko, xxii).

[4] Torczyner, Harry. (1977). *Magritte: Ideas and Images*. New York: H.N. Abrams, 71.

Figure 6 illustrates the fundamental distinction between classical sets and fuzzy sets. In the former (on the left), Magritte's famous pipe is either most certainly a pipe, or it is most certainly not. The boundaries of the crisp "This is a pipe" set are unambiguously defined; accordingly, an element, being either a pipe or not, is designated with the binary truth value of zero (for full exclusion) or unity (for full membership), and is consequently assigned membership constrained to the exhaustive set of {0,1} (Pedrycz, 4). Meanwhile, fuzzy sets (on the right of Figure 6) may give consideration to the somewhat paradoxical nature of Magritte's pipe; as a representation of a real pipe it possesses features thereof, and is therefore simultaneously both a pipe and not a pipe (and a member, to a degree, of both sets). While such a statement is inconsistent within classical set theory, the ambiguous boundaries of the fuzzy "This is a pipe" set allow Magritte's pipe ascription of a value in the interval [0,1], reflecting the degree to which it said to belong (Reveiz, 8).

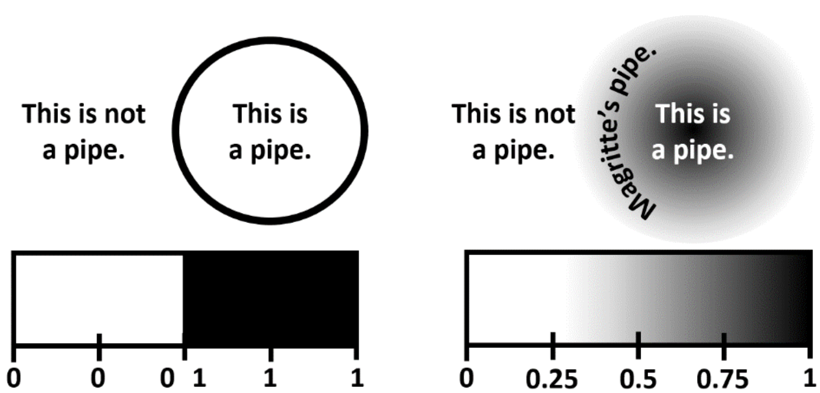


Figure 6. A Bivalent "Crisp" Set and a Multivalued "Fuzzy" Set.
Source: Author's elaboration based on Radionovs (2014).

Fuzzy logic also more closely resembles human logic than does classical probability theory; the techniques used to investigate fuzzy sets are as concerned with human psychology as they are mathematical formulations. In fact, fuzzy variables are often linguistic in nature; their values being words, not numbers. Fuzziness itself is largely a product of the imprecision of natural language and the distillation of mental abstraction into verbal representation. A commonly used example to demonstrate this point is that of rain, wherein "there are continuous gradations between overlapping linguistic categories: dense fog, drizzle, light rain, heavy rain, and downpour." Whether it is raining or not, in absolute terms, is an "extreme approximation" which implies that classical set theory is a special case of fuzzy sets (Kosko, xxii).

In the sense that natural language allows for "a little" or "a lot" of rain, the fuzziness of an event describes its ambiguity, or the degree to which it occurs. This is distinct from the randomness derived from an event's uncertainty of occurrence (it occurs or not) and answers the question of whether it is possible to "unambiguously distinguish the event from its opposite" (Kosko, 264). To this end, classical set theory inflexibly requires that the intersection of a set and its absolute complement is equal to the empty set. Bertrand Russell's *The Problems of Philosophy* (1912, 113) summarized this second of Aristotle's three traditional laws of thought, the "law of non-contradiction," by stating that "nothing can both be and not be."[5] Given the set A and its contradictory complement A^c,

$$A \cap A^c = \emptyset, \tag{2}$$

which represents the probabilistically impossible event

$$P(A \cap A^c) = P(\emptyset) = 0. \tag{3}$$

[5] The first of the "laws of thought" is the law of identity, or $A = A$. Bertrand Russell (1912) describes this as "whatever is, is." The identity principle is not addressed in this text, except, perhaps, insofar as the extension of de Morgan's laws to show the involutive nature of a set's complement; $(A^c)^c = A$. For Russell's mathematical representations, see Whitehead, A. N. & Russell, B. (1910). *Principa Mathematica*. Cambridge: University Press.

However, as demonstrated by Magritte's pipe in the discussion of Figure 6, "fuzziness begins when $A \cap A^c \neq \emptyset$." That is to say that set fuzziness only exists where the law of non-contradiction is violated (shown in Figure 7, this is the failure of the intersection of A and A^c to produce the empty set; a phenomena sometimes referred to as *overlap*). The third of the laws of thought is called the "law of excluded middle" and is defined as

$$A \cup A^c = \boldsymbol{X}, \tag{4}$$

which represents the probabilistically definite event

$$P(A \cup A^c) = P(\boldsymbol{X}) = 1, \tag{5}$$

wherein **X** denotes the sample space (or, in fuzzy logic, the "universe of discourse"). Bertrand Russell (1912) defines this as "everything must either be or not be."

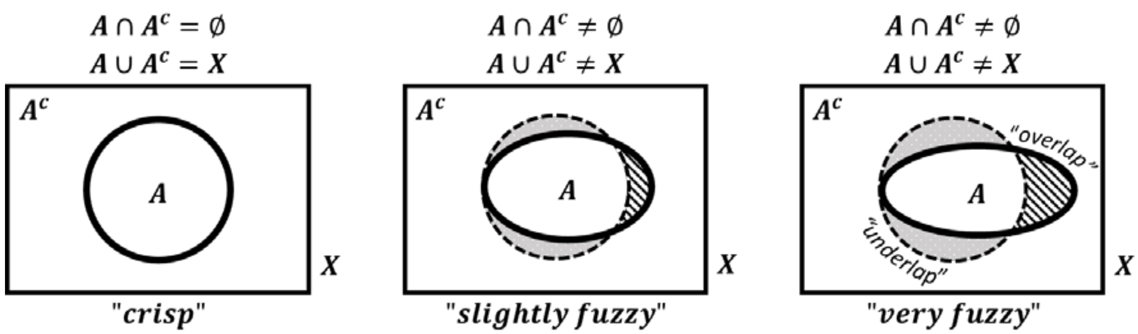


Figure 7. Elasticity and the Laws of Non-Contradiction and Excluded Middle.

The very measure of a set's fuzziness is determined by the extent to which the union of a set and its complement, $A \cup A^c$, "is a subset of its own subset $A \cap A^c$" (Kosko, 265). To clarify this point, fuzziness can be measured by the proportion of the union of complementary sets occupied by their intersection, which is depicted in Figure 7 and correspondingly represented by the "fuzzy" boundary illustrated in Figure 9. This is decidedly distinct from the proportion of the universe of discourse occupied by the intersection of complementary sets; fuzzy logic does not demand that the degrees of truth across all sets sum to unity for a specific object, effectively permitting violation of the law of excluded middle (represented in Figure 7 as the failure of the union of A and A^c to produce the universe, referred to as *underlap*). Fuzziness, then, occurs only when the laws of non-contradiction and excluded middle are unsatisfied as a result of "operation with membership values *between* 0 and 1," instead of exclusively 0 and 1, which is an otherwise paradoxical impossibility in classical set theory (Pedrycz, 38). It is, however, and while beyond the scope of this thesis, necessary to exercise caution as to not conflate fuzzy logic with contradiction-tolerant paraconsistent logics; while seemingly dialetheic, most fuzzy logics maintain truth-preservation in defining logical consequence as a matter of set ambiguity and are susceptible to deductive explosion; in practice, however, the evaluation of contradictory concepts is conducted with overlapping but distinct truth-retaining sets that negates this concern for approximate reasoning with vague information (Coniglio et al., 883).[6]

With respect to the matter of set ambiguity, Figure 7 also provides a visual illustration of elasticity; a concept that captures the essence of fuzziness as a matter of degree of truth. If one were to imagine the boundary containing set A as a rubber band, with the crisp depiction being that of the rubber band at rest, then the "slightly" and "very" fuzzy sets would represent forces applied to stretch the rubber band to their respective degrees. In this analogy, the amount of force required to sufficiently distort the rubber band

[6] The principle of explosion suggests that any asserted contradiction permits the logical inference of any given proposition. The subsequent cascade of inconsistencies trivializes the notion of truth.

(set A) as to contain an arbitrary element (x) initially existing outside of the space so encircled (set A^c), is inversely proportional to the element's degree of membership to, or conceptual compatibility with, set A. In this regard, propositions in classical logic are inelastic; given the proposition that "x belongs to A," the element x must satisfy the argument's predicate "belongs to A" in entirety, necessarily being perfectly classified as either true or false. Figure 7's latter two depictions may be thought of as instances where the predicate is satisfied, but only in part; for instance, in the case of "*slight*" fuzziness, an element x requiring the corresponding degree of "*slight*" elastic stretch for inclusion, might be considered to "*mostly* belong to A." These linguistic, imprecise, and vague predicates "appear very often in normal discourse, because they are very informative; common sense reasoning is elastic" (Trillas, 576).

2.4.1 Fuzzy Set Membership

Lotfi Zadeh (1965, 338) defines fuzzy sets as “a class of objects with a continuum of grades of membership.” So defined, the fuzzy set (class) A, as a subset of the universe of discourse $\boldsymbol{X}$ (whose variable x is a numerical value associated with the discourse of interest), possesses the characteristic function (herein called a membership function) $\mu_A(x)$ which relates, and is a mapping of, all values of $x \in \mathbf{X}$ to a real number in the interval [0,1]. The value of $\mu_A(x)$ at x is the “grade of membership,” or degree of truth, of x belonging to set A (Shapiro, 10).

$$\mu_{pipe}(x) = \begin{cases} 0 & for \quad x \leq 60 \\ 1 & for \quad x > 60 \end{cases}$$

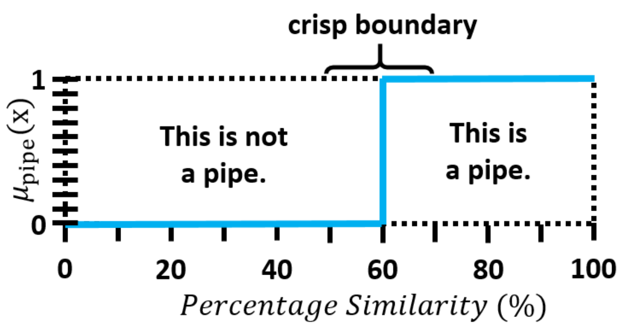


Figure 8. Crisp Set of Objects that are Pipes.
Source: Author’s elaboration based on Shapiro (2015).

Figure 8 and Figure 9 depict two membership functions, $\mu_{pipe}(x)$, representing both the crisp and fuzzy cases of the set “This is a pipe” which is defined by the psychometric scale measuring percentage in similarity of an object to a prototypical smoking pipe (alternatively, compatibility with the ideological concept of the pipe) (Cox, 91). In the figures, any object compared to the ideal pipe is assigned, through its respective membership function, a degree of truth in the interval [0,1]. The crisp boundary illustrated in Figure 8 is defined by a discrete function (a degenerate univariate) in which there is an unambiguous discontinuity that “jumps” at the defined threshold of 60%, prior to which the object is definitively not a pipe, and after which it is; accordingly, the membership values are constrained to the set {0,1}.

$$\mu_{pipe}(x) = \begin{cases} 0 & for \quad x \leq 50 \\ \dfrac{x-50}{20} & for \quad 50 < x \leq 70 \\ 1 & for \quad x > 70 \end{cases}$$

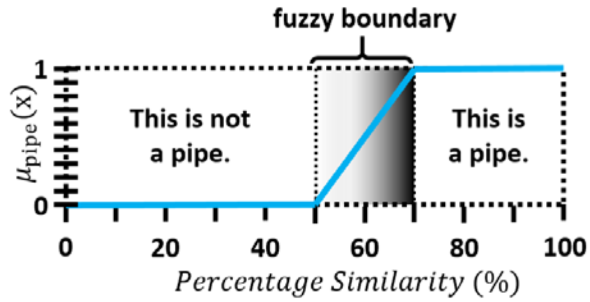
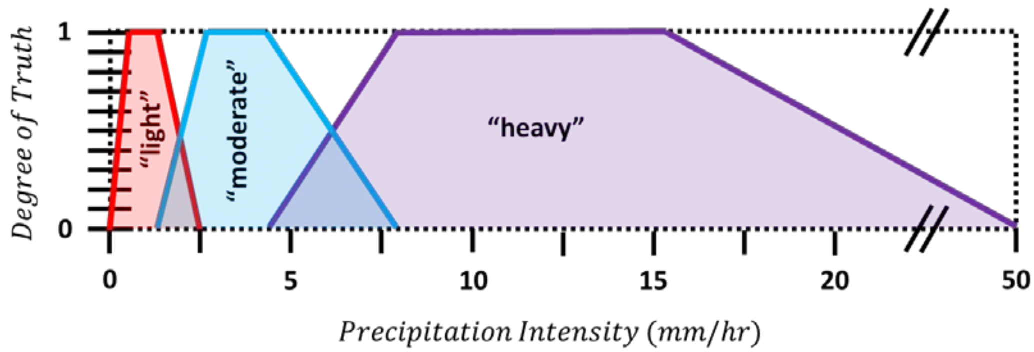


Figure 9. Fuzzy Set of Objects that are Pipes.
Source: Author's elaboration based on Shapiro (2015).

Particular to the fuzzy example in Figure 9, any objects determined to have a similarity of 70% or greater are assigned a degree of truth value of 1. Likewise, any objects possessing a similarity of 50% or less are assigned a degree of truth value of 0. It should be noted that these two values, 0 and 1, respectively imply either no or full membership in the "This is a pipe" set. This is equivalent to membership in the crisp classical set (as there exists no ambiguity in belonging). The ambiguity lies the boundary region between the similarity percentages of 50% and 70%; the membership function is an appropriately continuous piecewise linear "s," and constitutes a fuzzy set. For instance, it is trivial to observe, given the uniformly increasing membership function across the fuzzy range, that $\mu_{pipe}(60) = 0.5$. In linguistic terms, this is the case where an object possessing 60% similarity to the ideal pipe is assigned a degree of truth classification, or membership value, of 0.5.

While Figure 9 depicts the fuzziness internal to a single fuzzy set, it is often necessary to represent multiple fuzzy sets on the same universe of discourse in order to illustrate the ambiguity that exists between adjacent sets. Figure 10 portrays the fuzzy sets associated with “light,” “moderate,” and “heavy” intensities of precipitation in millimeters per hour as uniquely defined by the American Meteorological Society (AMS) and the Royal Meteorological Society (RMetS) according to their shared linguistic categorizations but divergent numerical descriptions.



$$\mu_{light}(x) = \begin{cases} 0 & for \quad x \leq 0 \\ \dfrac{x-0}{0.25} & for \quad 0 < x \leq 0.25 \\ 1 & for \quad 0.25 < x \leq 1 \\ \dfrac{2.5-x}{1.5} & for \quad 1 < x \leq 2.5 \\ 0 & for \quad x > 2.5 \end{cases}$$

$$\mu_{moderate}(x) = \begin{cases} 0 & for \quad x \leq 1 \\ \dfrac{x-1}{1.5} & for \quad 1 < x \leq 2.5 \\ 1 & for \quad 2.5 < x \leq 4 \\ \dfrac{7.6-x}{3.6} & for \quad 4 < x \leq 7.6 \\ 0 & for \quad x > 7.6 \end{cases}$$

$$\mu_{heavy}(x) = \begin{cases} 0 & for \quad x \leq 4 \\ \dfrac{x-4}{3.6} & for \quad 4 < x \leq 7.6 \\ 1 & for \quad 7.6 < x \leq 16 \\ \dfrac{50-x}{34} & for \quad 16 < x \leq 50 \\ 0 & for \quad x > 50 \end{cases}$$

Figure 10. Fuzzy Sets of Rainfall Rates.

In instances where the two organizations agree without question as to what should be included in a particular fuzzy set, the corresponding range of that numerically crisp rainfall rate is assigned a truth value of 1 (equivalently 0 when there exists no disagreement as to what should not be included). Nevertheless, the overlapping boundary regions are indicative of fuzziness resulting from inconsistent or contradictory definitions. One could readily speculate that the opposing characterizations of the same phenomena is due to a legitimate scientific disagreement in the meteorological community, that there exists a contributory statistical difference in observed North American and European rainfall rates, or that variance is inherent in the measurement of natural occurrences. It is, however, a genuine consideration that the RMetS model is granularized over and above that of the AMS'; the categories, though identically named, are therefore compressed relative to their American cousins. Although ostensibly a straightforward instantiation of fuzzy sets, this discussion illustrates many of the real-world drivers of ambiguity, particularly when linguistic categories are defined by expert opinion (or cross-organizationally).

A close examination of Figure 10's membership functions is also warranted. Certainly, given the context and available information, the trapezoidal is a credible candidate function (since the extreme minimum and maximum values are both well-defined and contain the unity membership in entirety). The shapes of the overlapping membership functions, and in particular the negative reciprocal slopes constituting the fuzzy boundary regions, suggest that for a given crisp input x, the truth value for one set is seemingly complimentary to that of its adjacent set. This is not universally true; truth degrees are not obligated to sum to unity, allowing non-complementary slopes that generate underlap.

It is also necessary, especially when dealing with mathematical operations on linguistic variables, to consider some other characteristics of fuzzy sets. It is first important to distinguish that a universe of discourse $\boldsymbol{X}$ is associated with the linguistic variable x

whose range defines the problem space which is subsequently decomposed into the individual overlapping fuzzy sets, each named with apparently self-descriptive terminologies appropriate to the variable's internal semantics. This taxonomy, as well as the fuzzy sets the terms represent, are collectively referred to as the *term set*, and are directly used in the logical construct of the ruleset by which some fuzzy models operate (Cox, 89). Figure 11 portrays several additional concepts used in describing such linguistic models. The *support* of fuzzy set A consists of all elements of $\boldsymbol{X}$ with a nonzero degree of truth,

$$Supp(A) = \{x \in \boldsymbol{X} | \mu_A(x) > 0\}. \tag{6}$$

The *core* of fuzzy set A consists of all elements of $\boldsymbol{X}$ that attain a membership degree of unity (accordingly, the *core* is inherently a subset of the *support*),

$$Core(A) = \{x \in \boldsymbol{X} | \mu_A(x) = 1\}. \tag{7}$$

Finally, an α-cut of fuzzy set A, or A_α, consists of all elements of $\boldsymbol{X}$ that attain a minimum membership degree exceeding the specified threshold α,

$$A_\alpha = \{x \in \boldsymbol{X} | \mu_A(x) \geq \alpha\}. \tag{8}$$

It is therefore evident that the *support* of A is equivalent to evaluating the α-cut at 0 and the *core* of A is defined where $\alpha = 1$ (Pedrycz, 14). Forcibly increasing the value of the α raises the threshold for set admittance, effectively determining a truth value at or below which membership should be considered zero for that particular application (Cox, 95).

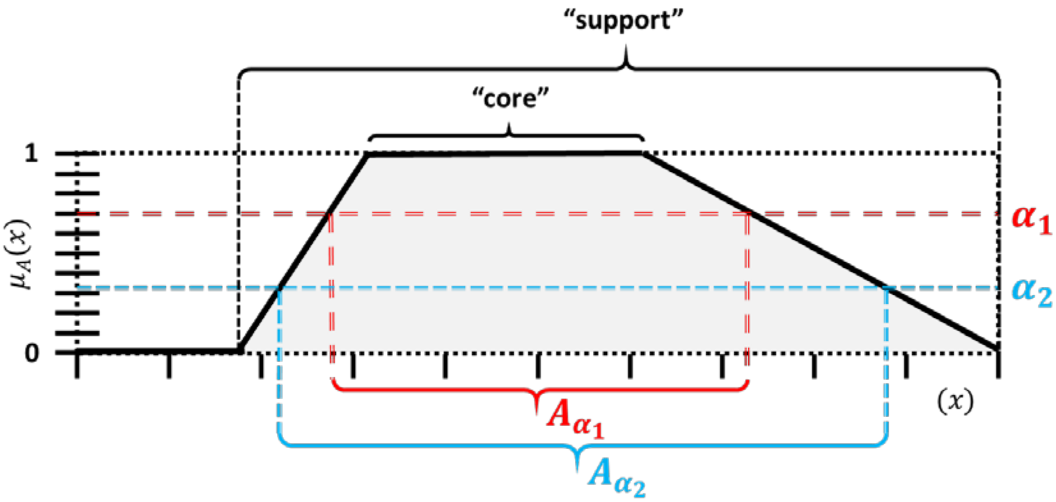


Figure 11. Support and $\boldsymbol{\alpha}$-Cuts.
Source: Author's elaboration based on Pedrycz (17).

Two α-cuts are considered in Figure 11, α_1 and α_2. Since more elements are admitted to α-cuts with lower α levels, it is the case that α_1 is a subset of α_2, or

$$if\ \alpha_1 > \alpha_2\ then\ A_{\alpha_1} \subset A_{\alpha_2}. \tag{9}$$

In this way, "any fuzzy set can be regarded as a family of fuzzy sets" wherein a fuzzy set can be constructed "from a family of nested sets (assuming that they satisfy the [above stated] constraint of consistency)" (Pedrycz, 17). In what is called the "representation theorem," any fuzzy set may therefore be decomposed into a family of subsumed α-cuts,

$$A = \bigcup_{\alpha \in [0,1]} (\alpha A_\alpha)\ or\ \mu_A \qquad \text{or, equivalently,} \qquad \mu_A(x) = \sup_{\alpha \in [0,1]} \left(\alpha A_\alpha(x)\right),$$

where "*sup*" is the set's supremum at the given α level. The importance of the representation theorem manifests in the implementation of fuzzy rules, the governance of interactions between multiple fuzzy sets, and its allowance of traditional mathematical techniques on fuzzy problem formulations. In effect, it permits the reconstruction of a set via the merger of its partial evaluations; this concept is fundamental to the aggregation of consequent fuzzy sets in inference systems, which will be introduced in Chapter III.

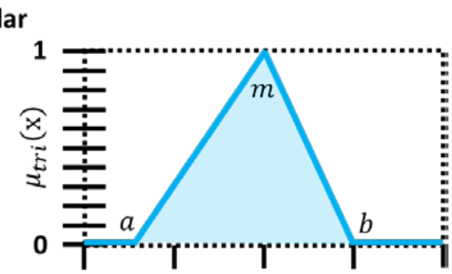
2.4.2 Membership Function Determination

In the discussion of the constituent membership functions depicting the several categorizations of rainfall in Figure 10, it is suggested that a trapezoidal shape is intuitively appropriate given the specific context of the problem as well as the limited information available (namely the categorical boundaries as presented by the two organizations, weighed equally). While the piecewise linear trapezoidal and its special case, the triangular membership function, are frequently used in the literature due to their simplicity in parameter estimation and low computational complexity, there are several other commonly encountered parametric functions. Among these standard parametric distributions are those illustrated in Figure 12. However, fuzzy membership functions may in fact take on any form that satisfies the mapping of the concerned concept, for input values over the specified universe of discourse, to output values between and including 0 and 1 (Pedrycz, 8); for fuzzy set A,

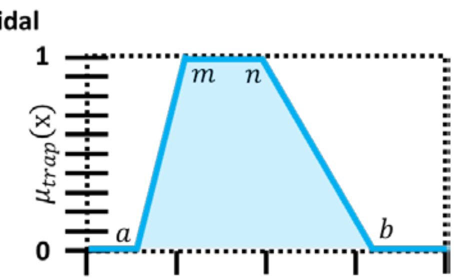
$$A: \boldsymbol{X} \rightarrow [0,1]. \tag{10}$$

In this sense, there exist no hard rules for the creation of membership functions, nor are there any "universal or pre-defined fuzzy sets;" the actual contours of a fuzzy set are entirely and exclusively representative of the semantic properties of the conceptual phenomenon in light of the model's context and outside of which the "fuzzy set has no meaning" (Cox, 100). A function's mathematical form and parameters, and consequently the latent knowledge it encodes, are subject to the intuition, experience, and information possessed by the formulator and their consulted decision-makers or subject matter experts. While nearly any shape is permissible, selection of membership functions should not be made arbitrarily; good models closely mimic real-world behaviors and the various classes of parametric distributions are often employed as adequate representations for particular classes of knowledge. Nevertheless, it is widely accepted that the "exact semantics captured by fuzzy sets is not too sensitive to variations in the shape" and are "tolerant of approximations" in both definition of the problem space and set representation (Pedrycz, 9; Cox, 100). While more complex functions, or their joint use, may better represent membership in real-world sets and possess higher information content and fidelity, the insensitivity of fuzzy models makes them quite robust to selection of membership function.

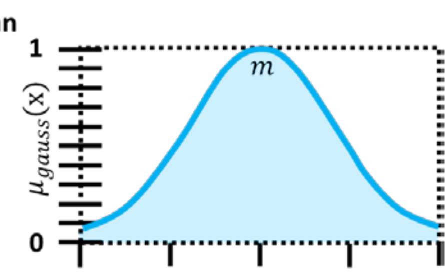
$$\mu_{tri}(x) = \begin{cases} 0 & for \quad x \le a \\ \dfrac{x-a}{m-a} & for \quad a < x \le m \\ \dfrac{b-x}{b-m} & for \quad m < x \le b \\ 0 & for \quad x > b \end{cases}$$



$$\mu_{trap}(x) = \begin{cases} 0 & for \quad x \le a \\ \dfrac{x-a}{m-a} & for \quad a < x \le m \\ 1 & for \quad m < x \le n \\ \dfrac{b-x}{b-n} & for \quad n < x \le b \\ 0 & for \quad x > b \end{cases}$$



$$\mu_{gauss}(x) = e^{-k(x-m)^2} \qquad where \quad k > 0$$



$$\mu_{sig}(x) = \begin{cases} 0 & for \quad x \le a \\ 2\left(\dfrac{x-a}{b-a}\right)^2 & for \quad a < x \le m \\ 1 - 2\left(\dfrac{x-b}{b-a}\right)^2 & for \quad m < x \le b \\ 1 & for \quad x > b \end{cases}$$

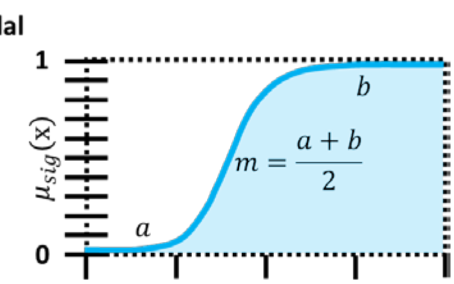


Figure 12. Common Membership Functions.

The parameterized functions presented in Figure 12 are generally representative of two encompassing types of fuzzy sets, *fuzzy numbers* and *fuzzy qualifiers*. In each depiction and for all nonnegative values of membership, the parameter m is the modal value, a is the lower bound, and b is the upper bound. Triangular and bell-shaped curves, akin to Figure 12's triangular and Gaussian functions, are often used to represent the quantitative approximation of a specific numeric value (any similarly convex distributions like some exponential, hyperbolic, or normal functions, among others, are contextually viable). Again in Figure 12, only the single-point supremum of the triangular and Gaussian achieves a membership degree of 1, and can consequently be considered to depict the fuzzy set approximating the central value m. In what are sometimes called *fuzzy numbers*, the kurtosis (or, in the triangular case, spread) of such functions may be indicative of the degree to which the approximation is precise (whether the approximation is accurate is a different question entirely)[7]. While bell-shaped membership functions are favored for their concise notation, consistent smoothness, and for maintaining nonzero values across the universe of discourse (since they only asymptotically approach 0), triangular distributions are often satisfactory surrogates due to their low information and calculational demands, especially in light of the aforementioned insensitivity of fuzzy systems. Nevertheless, it is informative to consider the properties and typical behaviors modeled by common probability distributions when constructing fuzzy membership functions; Law (2007, 275) presents an extensive treatment on probability distribution functions for simulation input.

It is also obvious when comparing Figure 12's membership functions that the triangular is identical to the trapezoidal when m and n are coincident. Trapezoidal or platykurtic bell-shaped functions are often appropriate when representing classes of numbers or conceptual categorizations. Many of these functions are structured with a

[7] Kurtosis is the fourth moment about the mean of a probability distribution and is reflective of its "tailedness." Platykurtic distributions have thin tails and are generally described as "fat" or "flat-topped" and may therefore indicate a broader, but more uniform, approximation. Leptokurtic distributions have fat tails and descriptively have more "peakedness," representing a narrower but more extreme approximation (Cox, 521).

plateau designating full membership over a specified numeric interval. Conversely, *fuzzy qualifiers* are used to model concepts exhibiting asymmetric, dichotomous, or unbounded behaviors. The sigmoidal contour in Figure 12, or any comparable "S-curve," polynomial, logistic, or strictly linear representation typically attains full membership at its open-ended side, whether left or right, and possess a zero degree of membership on the closed side. These upper- and lower-bound parameters, and importantly the inflection point m, are selected to reflect the suspected or known distribution of the population of interest, as well as its underlying characteristics (Cox, 112). Specifically, S-curves regularly correspond with the growth curves of continuous random variables and their cumulative distribution functions. In this regard, they are also suitable when dealing with event frequencies, time-series, proportional dependencies, and imprecision in conditional qualifications; however, fuzzy propositions "involving 'usuality' terms lead to a class of ultra-fuzzy implications [in fuzzy reasoning]" (Cox, 114).

2.4.3 Fuzzy Set Operations

In order to perform mathematical operations to combine, compare, or otherwise aggregate fuzzy sets, it is necessary to extend propositional logic from classical bivalence to one of multivalued logic. Having already established that classical sets are a special case (equivalently, a subset) of fuzzy sets in which truth values are anchored to the extremes of 0 (absolute exclusion) and 1 (absolute inclusion), the basic logical connectives of conjunction (and), disjunction (or), and negation (not) are preserved under conditions of multivalence. Furthermore, because membership functions are "equivalent representations of sets," the mathematical operators of intersection ($\cap$), union ($\cup$), and complement (A^c), which are in turn equivalent to the respective logical connectives, are correctly represented by evaluating the "minimum, maximum, and one-complement of the corresponding [membership] functions for all $x \in \boldsymbol{X}$:"

$$(\mu_{A\cap B})(x) = min\big(\mu_A(x), \mu_B(x)\big) = \mu_A(x) \wedge \mu_B(x),$$
$$(\mu_{A\cup B})(x) = max\big(\mu_A(x), \mu_B(x)\big) = \mu_A(x) \vee \mu_B(x), \tag{11}$$
$$\mu_A^c(x) = 1 - \mu_A(x),$$

wherein A and B are defined sets in the universe of discourse $\boldsymbol{X}$ and $\mu_{A\cap B}$ and $\mu_{A\cup B}$ are the membership functions resulting from A and B's intersection and union, respectively (Pedrycz, 31). Use of the min ($\wedge$), max ($\vee$), and additive complement $(1 -)$ operators permit application of the logic to continuous sets; that is, they both satisfy preservation of the truth values according to bivalent logic while simultaneously allowing real numbers between 0 and 1 (Reveiz, 12). Utilizing the analogy and membership functions introduced in Figure 10, a comparison of bivalent logical operators and multivalent logical operators in assessing several propositional constructs is illustrated in Figure 13.

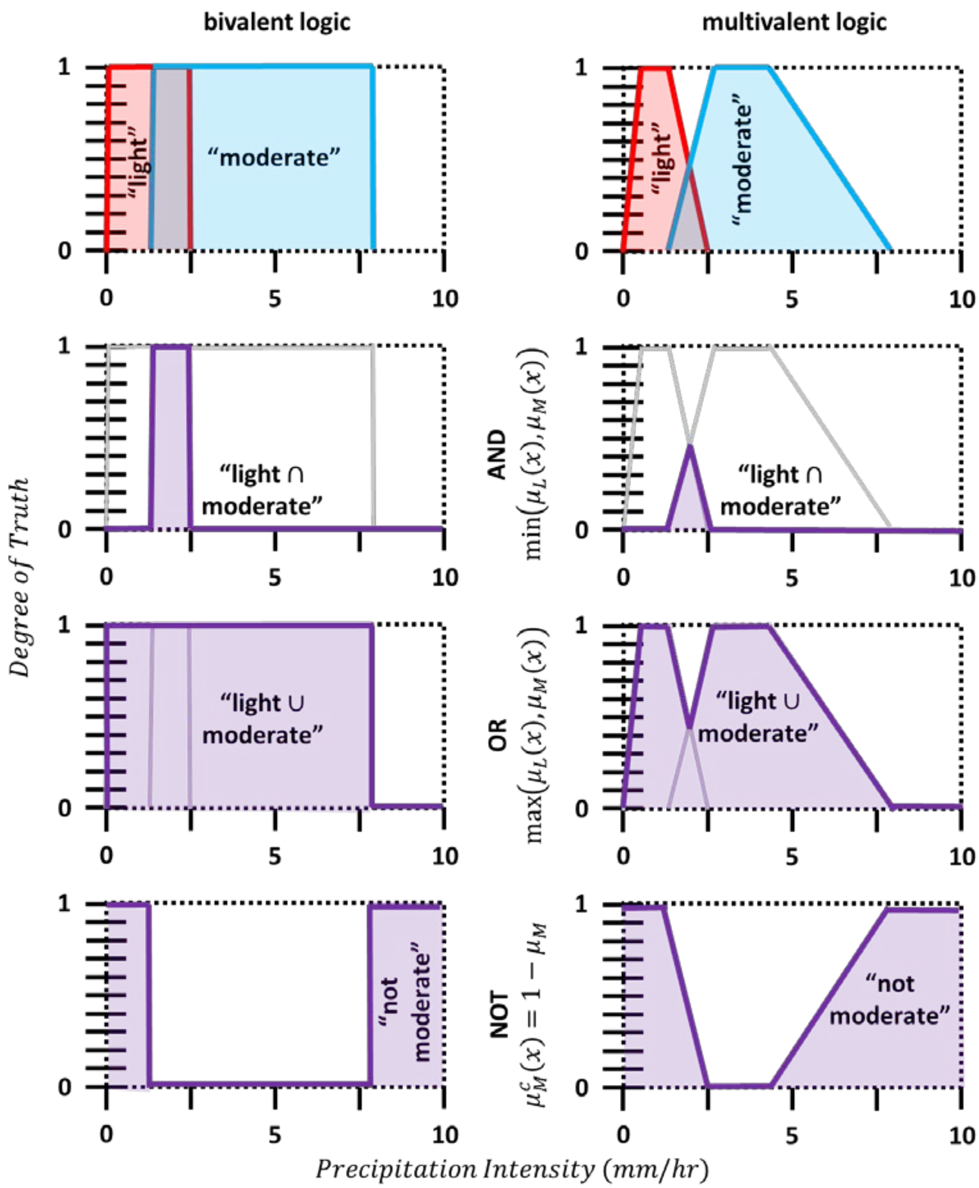


Figure 13. Logic Operators under Bi- and Multivalence.
Source: Author's elaboration based on Mathworks (2018).

However, the *min*, *max*, and additive complement mathematical functions do not provide a singularly unique definition of the logical operations in both bivalent and multivalent logics; while most applications of fuzzy logic adopt these operations, they represent only a particular correspondence between the two systems of logic. In fact, the specific operations used to define fuzzy conjunctions and disjunctions are "arbitrary to a surprising degree," and many alternative functions have practical use (Mathworks, 1-21). In general, the intersection (logical conjunction) of two fuzzy sets A and B is specified by the binary algebraic operation T, or *t-norm*, used to aggregate their respective membership functions such that for all $x \in \boldsymbol{X}$,

$$(\mu_{A \cap B})(x) = T\big(\mu_A(x), \mu_B(x)\big), \tag{12}$$

which performs the binary mapping

$$T: [0,1] \times [0,1] \rightarrow [0,1]. \tag{13}$$

The *t-norm*, or triangular norm, is adopted from probabilistic metric spaces which requires a generalization of the triangle inequality (in geometry, that the sum length of any two sides of a triangle must be greater than or equal to the length of the third) of ordinary metric spaces subjected to probability theory, such that distances are characterized by probability distributions (Menger, 536). In fuzzy set theory, triangular norms form the basis for mathematical operations on fuzzy sets, and must satisfy the following basic properties, including boundary conditions such that they behave correctly as a generalization of set operations on crisp sets (Pedrycz, 33):

- Commutativity: $T(x, y) = T(y, x)$,
- Associativity: $T(x, T(y, z)) = T(T(x, y), z)$,
- Monotonicity: $if\ x \leq y\ and\ w \leq z, then\ T(x, w) \leq T(y, z)$,
- Boundaries: $T(0, x) = 0,\ T(x, 1) = T(1, x) = x$.

As with logical conjunctions in classical bivalence, the commutative (indifference to order of membership function aggregation in conjunction) and associative properties (indifference to order in pairwise conjunction of any number of membership functions) hold in multivalence. Likewise, monotonicity dictates, for example, that an increase in the truth or membership values of conjuncts is prohibitive of a decrease in the truth or membership value of the corresponding conjunction. Lastly, in the case of boundary conditions, inclusion of the identity element (1) implies the constraining extremes of multivalence; a truth or membership value in the bivalent set $\{0,1\}$ represents *false* and *true* assessments, respectively.

Correspondingly, the union (logical disjunction) of two fuzzy sets A and B is specified by the binary algebraic operation S, called the *t-conorm* or *s-norm*, used to aggregate their respective membership functions such that for all $x \in \boldsymbol{X}$,

$$(\mu_{A \cup B})(x) = S\big(\mu_A(x), \mu_B(x)\big), \tag{14}$$

which performs the binary mapping

$$S: [0,1] \times [0,1] \to [0,1]. \tag{15}$$

The *s-norm*, or triangular co-norm, is formally the dual of the *t-norm* and, commensurate with de Morgan's laws[8], is complementary to any given *t-norm* T by way of negation,

$$S(x, y) = 1 - T(1 - x, 1 - y). \tag{16}$$

[8] de Morgan's laws state that the complement of a union is equivalent to the intersection of complements where $(A \cup B)^c = A^c \cap B^c$ and the complement of an intersection is equivalent to the union of complements where $(A \cap B)^c = A^c \cup B^c$. Weisstein, Eric W. (2018). "de Morgan's Laws." *MathWorld*. http://mathworld.wolfram.com/deMorgansLaws.html.

Like their *t-norm* duals, *s-norms* must satisfy the following axiomatic properties:

- Commutativity: $S(x, y) = S(y, x)$,
- Associativity: $S(x, S(y, z)) = S(S(x, y), z)$,
- Monotonicity: $if\ x \leq y\ and\ w \leq z, then\ S(x, w) \leq S(y, z)$,
- Boundaries: $S(x, 0) = S(0, x) = x,\ S(x, 1) = 1$.

Subject to the defining axioms, triangular norms and co-norms therefore define general classes of operators for assessing the intersection and union of fuzzy sets, and many parameterized *t-norms* and *s-norms* are common in the literature. Pedrycz and Gomide (1998, 33) present a substantial treatment of those most frequently encountered, as well as a discussion of specific features of their subclasses. While each unique mapping "provides a way to vary the gain on the function so that it can be very restrictive or very permissive," it is evident when considering the boundary conditions established by the basic properties of triangular norms and their co-norms that the min ($\wedge$) and max ($\vee$) operators respectively belong to the classes of *t-norms* and *s-norms*, and are perhaps the most frequently used in practice (Pedrycz, 33; Mathworks, 1-23). Not coincidentally, these functions are in fact those utilized by Kurt Gödel's (1932, 65) intuitionistic multi- and infinitely-valued logics shown to be completely sufficient for axiomatization. Figure 14 depicts the three dimensional and contour graphs of several common mappings, encapsulated by, and as an interval-constrained instance of, the Schweizer-Sklar family of triangular norms (1963, 69).

The minimum *t-norm*, when constrained to bivalence, corresponds to the set intersection operator and therefore provides an upper bound on the *t-norms* class (whose supremum is 1). Alternatively, the *ordinary product* is sometimes used for intersection (as it is with the probabilities of two independent events in probability theory; $P(A \cap B) = P(A)P(B)$), whereas the lower bound on the *t-norm* class is formed by the *drastic product*. Accordingly, the bounds of *t-norms* are defined by

$$T_{drastic}(x,y) \leq T(x,y) \leq T_{min}(x,y). \tag{17}$$

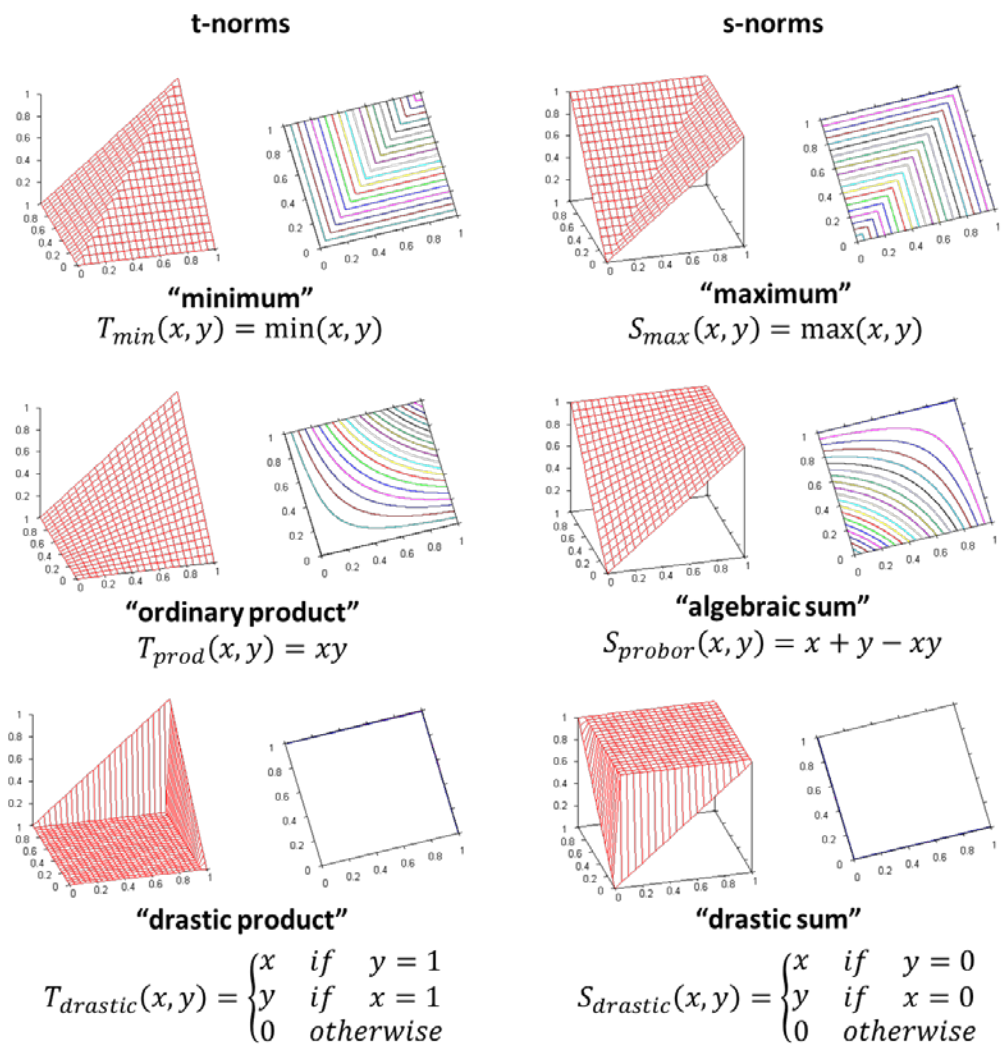


Figure 14. Common and Boundary Triangular Norms and Co-Norms.
Source: Author's elaboration based on Béhounek, Libor. (2007). Public Domain.

Likewise, the maximum *s-norm*, as the dual to the minimum *t-norm*, constrained to bivalence corresponds to the set union operator and provides the lower bound on the *s-norm* class (whose infimum is 0). Similar to its dual, the *algebraic sum* (here, $probor$) is sometimes used for union (again, in probability theory, that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$), and the lower bound on the *s-norm* class is formed by the *drastic sum*. Accordingly, the bounds of *s-norms* are defined by

$$S_{max}(x, y) \leq S(x, y) \leq S_{drastic}(x, y). \tag{18}$$

III. Methodology

"It has been observed, that a weight A of 10 grams and a weight B of 11 grams produce identical sensations, that the weight B is just as indistinguishable from a weight C of 12 grams, but that the weight A is easily distinguished from the weight C. Thus the raw results of experience may be expressed by the following relations: A=B, B=C, A<C, which may be regarded as the formula of the physical continuum."[9]

- Henri Poincaré
La Science et l'Hypothèse (1902, 34)

3.1 Chapter Overview

This chapter presents a general methodology for the quantification of military operational risk that coincides with the process of *Risk Appraisal* as defined in the DoD's Joint Risk Analysis Methodology. This general methodology capitalizes on the human-thought-like approximate reasoning afforded by fuzzy logic by way of a fuzzy inference system, introduced in Section 3.2. The first phase of the method, knowledge elicitation, is discussed in Sections 3.3.1 through 3.3.3, and navigates the process of building the model, particularly in the context of Joint Planning (JP 5-0) and Joint Operations (JP 3-0). Sections 3.4.1 through 3.4.6 discuss the mathematical mechanisms of the model's execution. Section 3.5 concludes the chapter and discusses use of the model's results. For exposition, a small-scale example problem parallels the sequential processes examined throughout.

[9] Poincaré, Henri. (1913). *Science and Hypothesis*. Trans. Halsted, George Bruce. New York: The Science Press. Poincaré is referring to the observations of Gustav Fechner (1801-1887), who in 1860 published *Elemente der Psychophysik*; the Weber-Fechner laws postulate the differences between actual and perceived physical stimuli. In essence, the inherent inaccuracy of human sensory perception in the physical continuum necessitates approximation that permits relative comparison, but not strict distinction, of physical phenomena.

3.2 Fuzzy Inference Methodology

The proposed methodology for reconciling current operational risk management practices with their apparent deficiencies utilizes the same fundamental rule-based structure employed in the logic controllers of many automated devices, a process referred to as a fuzzy inference system (FIS), fuzzy logic inference system (FLIS), or fuzzy expert system (FES) (Shapiro, 17). In these processes, crisp input values are ultimately mapped to a crisp output space through a series of fuzzy operations independently detailed in Chapter II. In the specific context of military operational risk management, it is desirable that the input variables be metrics correlated with suitable key risk indicators (KRIs); the output value is an aggregate risk value used to inform risk decisions or course of action comparison in operational planning (Girling, 251). Illustrated in Figure 15, a FIS can be thought to consist of two primary structures; first, a knowledge base in which the risks, their indicators, and measures inform the construction of a membership function database and compatible logical rule base. Second, a logical processor, or "inference engine," that subjects the measured (or projected) input variables to a procedure consisting of the five sequential subprocesses of fuzzification, composition, implication, aggregation, and defuzzification. While the exact architecture of the FIS presented in this document is a commonly practiced one, its unique application within the DoD's existing risk framework warrants, for the purpose of brevity, the discrimination between it and the general case, and is henceforth referred to as the "Risk Appraisal Fuzzy Inference System," or RAFIS.

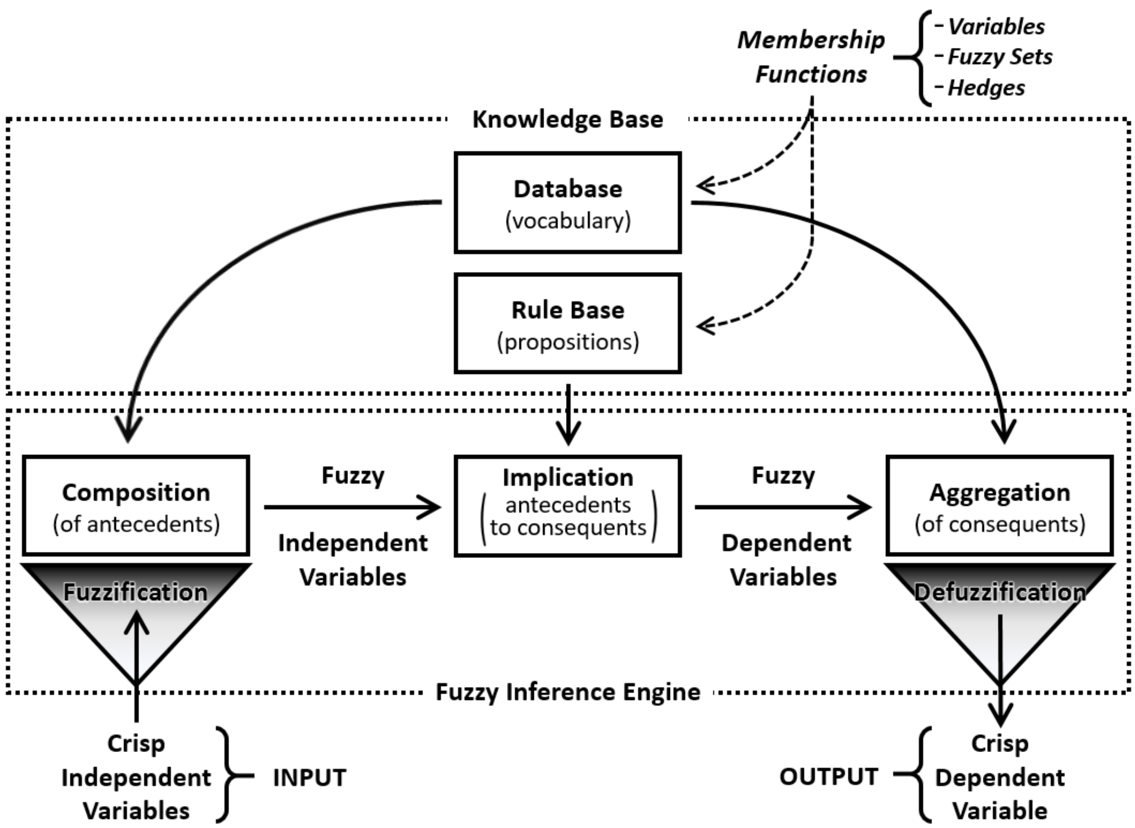


Figure 15. Anatomy of a Fuzzy Inference System.
Source: Author's elaboration based on Shapiro (18) and Reveiz (17).

The Joint Risk Analysis Methodology (JRAM) detailed in Chapter II describes a framework consisting of three fundamental pillars and four supporting activities. Of these components, the RAFIS methodology proposed herein is concerned primarily with *Risk Appraisal*, "the generation of knowledge and understanding," as opposed to *Risk Management* which entails the actualization of risk decisions and implementation of controls (JRA, B-2). Naturally, effective *Risk Communication* is a persistent requirement throughout the risk analysis process; as an expert system, it is in the nature of any FIS to facilitate inter-domain conversation, particularly in the collaborative design of the logical rule base. Additionally, the proposed model's knowledge elicitation process abstracts a risk ontology that satisfies the JRAM's subordinate steps of *Problem Framing* and *Risk Assessment* which jointly require specification of the military operation to be modeled, examination of its particular risks, exploration of their correlations and causal pathways, development of measurement criteria, and definition of a problem specific vocabulary. Finally, the first element of *Risk Judgment* is addressed through the crisp numerical output of the RAFIS; it is the goal of *Risk Characterization* to capture comparative assessments of the operation's independent risk factors. While admitting that quantification and visual depiction are desirable when informing risk decisions, and accordingly endorsing mathematical expectation and the "risk [matrix] contour graph" as tools thereof, the JRAM concedes that it "is ultimately a *qualitative* effort" (JRA, B-4).

Alternatively, the RAFIS offers a methodology that consistently provides *quantitative* valuations of risks and necessarily generates, as a byproduct of the inference process, 'fuzzy risk matrices' for the pictographic comparison of all pairwise indicators and their resultant risk values. The RAFIS, embedded and subordinated to the JRAM and as a continuously iterative parallel to the Joint Planning Process (JPP), is depicted in Figure 16 as the motivating apparatus that energizes the wholesale risk appraisal and management construct; appropriate to this analogy, the Fuzzy Inference Engine is, in particular, the driving mechanism in the conveyance of risk knowledge, understanding, decision, and action.

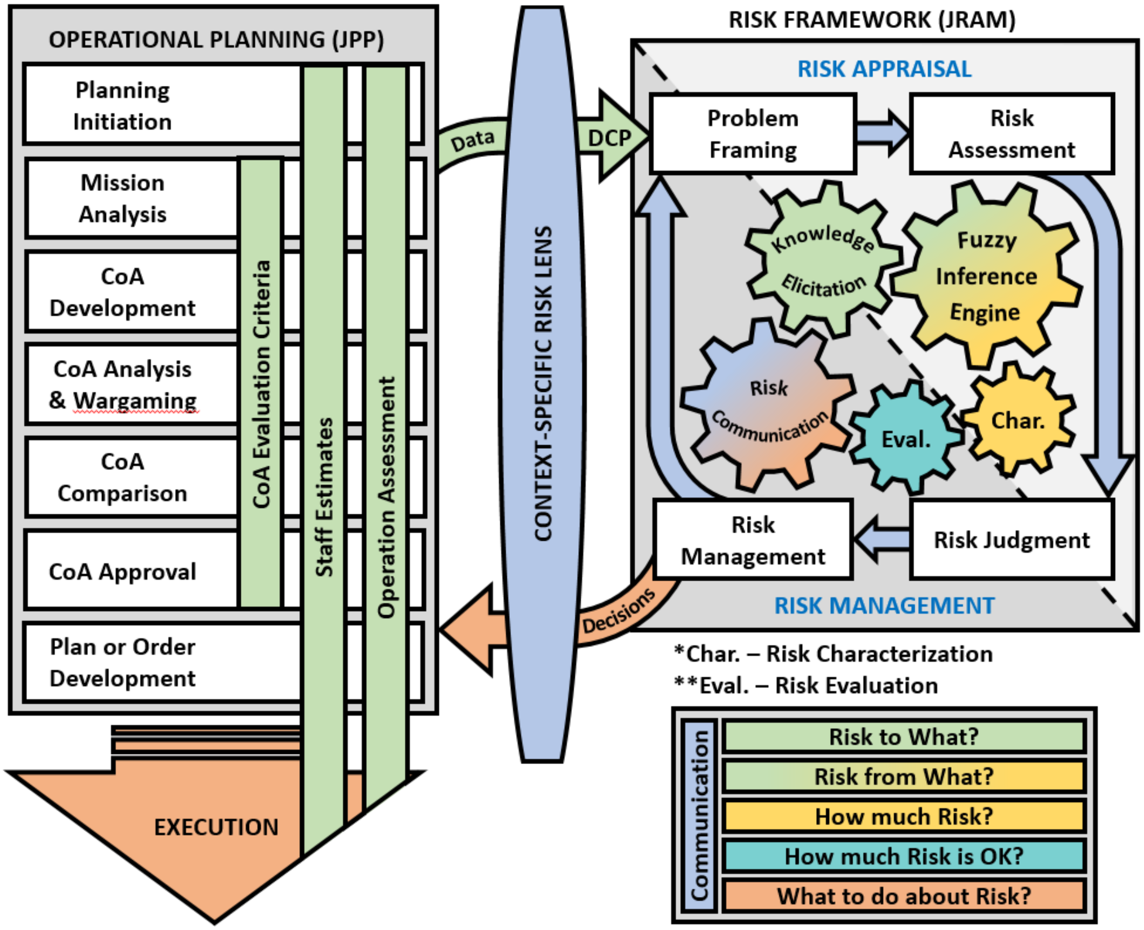


Figure 16. Dynamics of the JPP, JRAM, and RAFIS.

3.3 Knowledge Elicitation

In what is most closely synonymous with the JRAM's *Problem Framing*, but also concerned with *Risk Assessment*, knowledge elicitation is the process by which the model's mathematical structure is formulated and its constituent elements parameterized. As operational risks are, in large part, comprised of emerging threats in diverse geographic environments, they are expectedly subject to underdeveloped experience data. Consequently, expert opinion is elicited to serve as the principal evidentiary source from which the model's construction is informed; in particular, the causal relationships specified in the inference system's rule base confer meaning and utility to the otherwise unenlightened model. It is essential that the analyst, serving as the knowledge engineer, exercise due diligence in facilitating elicitation; the very quality of the advising experts' opinions directly translates to the model's credibility and usefulness. In keeping with the JRAM, knowledge elicitation is a continuous and iterative process in which feedback mechanisms are established to fine-tune the system's parameters and rules. Accordingly, the RAFIS capitalizes on three information vectors contemporaneously present throughout the JPP and illustrated in Figure 16: course of action (CoA) evaluation criteria, staff estimates, and operation assessment. While risks and their indicators are not fully coterminous with the standards and measures established in each of these distinct veins, they form a repository from which the knowledge base may, in part, be extrapolated.

CoA evaluation criteria are designed to measure a CoA's relative effectiveness by distinguishing the contributory factors of mission success from those of mission failure (and in accordance with the commander's planning guidance). Established prior to wargaming as a hedge against bias and subjectivity when testing, they are precisely defined and often evaluated with a numerical score by the staff member with functional area responsibility and weighted according to relative importance. It is plainly evident how this process might contribute to identifying risk factors and their indicators, bounding possible criterion domains, parameterizing result categories and, ultimately, through wargaming, provide suitable estimates for model inputs. Appendix G of JP 5-0, Joint Planning, offers an overview and example of CoA comparison techniques.

Staff estimates, meanwhile, provide a running and more highly-granulized assessment of a functional area's level of mission support. Again, initial estimates are expected to assist in structuring the RAFIS; identification of essential resource shortfalls, capability limitations, or operational impediments may prove to be among the dominant drivers of mission risk. Subsequent updates naturally allow for model refinement, not only up to the point of input variable generation for the purposes of CoA comparison, but also in monitoring the evolution of risk as the operation proceeds in execution. With regard to external adversarial threats or Operational Environment (OE) hazards, it is of particular consequence that "critical knowledge gaps in initial estimative intelligence" and the validation of key planning assumptions be addressed through a comprehensive intelligence collection plan (JP 5-0, V-16). In many cases, the importance of the associated risks necessitates the establishment of Priority Intelligence Requirements (PIRs) specific to their causal factors or indicators; for this reason, bilateral communication between the intelligence staff and the knowledge engineer is imperative. In competing for the prioritization of limited collection assets, the risk analyst must advocate when necessary, but also be critically-minded in determining the true merit of the KRI as an indicator of its associated risk factor. Appendix C of JP 5-0, Joint Planning, details the process of capturing staff estimates in the context of the JPP.

Finally, operation assessment "refers specifically [to measuring] progress [or regression] towards accomplishing tasks, creating conditions or effects, and achieving objectives" during both planning and execution (JP 3-0, II-9). In this capacity, it utilizes Measures of Effectiveness (MOEs) to monitor the degree of change in the OE due to an operational task; Measures of Performance (MOPs) are designed to evaluate the standard to which the task is conducted. While staff estimates are a natural vehicle for these measures, operation assessment is distinctively oriented on the linkages between an action and its desired effect, or attainment of an end state. Indicators like MOEs and MOPs are judiciously selected for their ability to delineate causalities and are consequently subjected to a number of axiomatic efficacy gauges that are expectedly congruent with those applied to Key Risk Indicators. While Annex A to Appendix D of JP 5-0, Joint Planning, provides a satisfactory overview of operation assessment, it is the Data Collection Plan (DCP) discussed in Annex B that is of most interest to the knowledge engineer. The DCP institutes a number of additional criteria to ensure the 'measurability' and methods of MOP and MOE indicator collection; these criteria are equally applicable to KRIs (JP 5-0, D-B-1).

3.3.1 Determine Risk Hierarchy, Risk Factors, and Key Risk Indicators

$$
\begin{aligned}
&RF_f\text{: Set of Risk Factors indexed by } f \in \{1, 2, \ldots, m\} \\
&\boldsymbol{Y}_f\text{: Universe of Discourse for Risk Factor } f \\
&\boldsymbol{y}\text{: State Vector of Dependent Variables, } \boldsymbol{y} = (y_1, y_2, y_f, \ldots, y_m) \\
&LV_{RF,f}\text{: Term Set for Risk Factor } f \\
&KRI_k\text{: Set of Key Risk Indicators indexed by } k \in \{1, 2, \ldots, n\} \\
&\boldsymbol{X}_k\text{: Universe of Discourse for Key Risk Indicator } k \\
&\boldsymbol{x}\text{: State Vector of Independent Variables, } \boldsymbol{x} = (x_1, x_2, x_k, \ldots, x_n) \\
&LV_{KRI,k}\text{: Term Set for Key Risk Indicator } k
\end{aligned}
\tag{19}
$$

This first component of knowledge elicitation answers, to an extent, the "risk *to* what?" and "risk *from* what?" questions posed in the *Problem Framing* and *Risk Assessment* activities. Taken together, these two questions define the information desired as output from the model (*to* what?) and the informational demands of the model's input (*from* what?) (Cox, 543). Whereas the JRAM incorporates the quantification of scales, probabilities, and consequences in these activities, the RAFIS does not. Instead, the output of this process is a hierarchical vocabulary of the linguistic universes, sets, and variables that comprise the scenario's risk dialogue. The "context-specific risk lens" in Figure 16 alludes to the exacting inspection demanded of the knowledge engineer in circumstantial contemplation of the military operation of interest and its accordant compulsory tailoring of the risk framework. Specifically, the risk analyst must extricate, from the JPP's three information vectors, a modeling infrastructure that is contextually considerate of the problem; there is no universal ontology for military operational risk. Indeed,

> *"the military instrument of national power can be used in a variety of ways that vary in purpose, scale, risk, and combat intensity [and can be] understood to occur across a continuum of conflict ranging from war to peace"* (JP 1, xi).

While not prescriptive, it is useful to consider a variety of systems perspectives, or "lenses," for decomposition of the OE, across the range of military operations, into a manageable construct. Among these are:

- Operational Variables (PMESII-PT); Political, Military, Economic, Social, Infrastructure, Informational, Physical Environment, Time (JP 3-0, IV-3),

- Mission Variables (METT-TC); Mission, Enemy, Terrain and Weather, Troops and Support Available, Time Available, Civil Considerations (ADRP 3-0, 1-2),

- ASCOPE; Areas, Structures, Capabilities, Organizations, People, Events (JP 5-0, IV-11),

- Principles of Joint Operations (Principles of War); Objective, Offensive, Mass, Maneuver, Economy of Force, Unity of Command, Security, Surprise, Simplicity, Restraint, Perseverance, Legitimacy (JP 3-0, I-2),

- Joint Functions (Warfighting Functions); Command and Control, Intelligence, Fires, Movement and Maneuver, Protection, Sustainment (JP 3-0, III-1),

- Operational Environment Dimensions; Air, Land, Maritime, and Space Domains, the Electromagnetic Spectrum, Information Environment (including Cyberspace) (JP 5-0, IV-10),

- (5-M); Man, Machine, Medium, Management, Mission (AFPAM 90-803, 13),

- Joint Capability Requirements (DOTmLPF-P); Doctrine, Organization, Training, material, Leadership and Education, Personnel, Facilities, Policy (CJCSI 5123.01H, A-8).

The RAFIS is structured as a three tiered hierarchy in which the comprehensive Operational Risk is expanded into a set of m subordinate Risk Factors (RF_f) that constitute, and are in fact synonymous with, the model's consequent (dependent variables, y_f) universes of discourse ($\boldsymbol{Y}_f$), and could take the form of "risk *to*…" the various elements of the systems perspectives enumerated in the preceding paragraph. The Risk Factors are further expanded into n Key Risk Indicators (KRI_k) that are either the direct determinants or adequately proximate indicators of the prime drivers of the Risk Factors, which comprise, and are likewise synonymous with, the model's antecedent (independent variables, x_k) universes of discourse ($\boldsymbol{X}_k$). In this manner, the KRIs are representative of "risk *from* what?" Each Risk Factor and KRI are then further decomposed into term sets, effectively quantizing the linguistic variables into subordinate taxonomies of fuzzy sets

($LV_{RF,f}$ and $LV_{KRI,k}$, respectively). The number of fuzzy set 'terms' internal to each term set is an important characteristic of the model's semantics that allows for varying degrees of granularity in expert judgment, but should be considered with regard to the shape and overlap of membership functions determined in the following elicitation step. Figure 17 depicts the general structure of a hierarchy developed by this process.

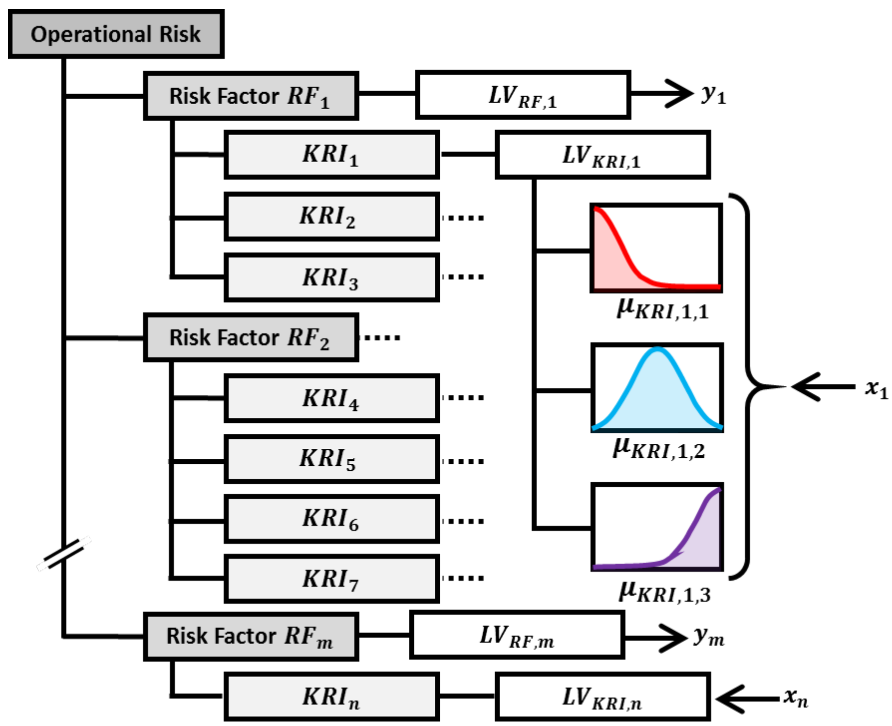


Figure 17. RAFIS Risk Hierarchy.

While not mathematically necessary to differentiate a Risk Factor or KRI from its universe (for instance, KRI_k and $\boldsymbol{X}_k$), the distinction is made in the RAFIS to isolate linguistic variables from their quantitative descriptions. Similarly, a term set need only specify its linguistic elements; the fuzzy membership function defining each elemental term is calibrated later in the elicitation process. To clarify the outcome of this process, a trivial example is introduced in which the problem of interest is adequately addressed by a single risk factor. It is therefore termed, simply, *Risk Level*, where

$$RF_1 = Risk\ Level. \tag{20}$$

Contributing to this Risk Factor are the two KRIs elicited from expert opinion, *Probability* and *Severity*,

$$KRI_k = \{Probability, Severity\}. \tag{21}$$

In the example, suppose that expert opinion suggests that an appropriate resolution for quantizing *Risk*, *Probability*, and *Severity* is by three categories each. The term sets are, respectively,

$$\begin{gathered} LV_{RF,1} = \{Low, Medium, High\}, \\ LV_{KRI,1} = \{Unlikely, Occasional, Frequent\}, \\ LV_{KRI,2} = \{Negligable, Moderate, Critical\}. \end{gathered} \tag{22}$$

These definitions collectively satisfy the output of this step for the given example. The subsequent step involves parameterization of the universes of *Probability*, *Severity*, and *Risk Level* and membership functions giving mathematical description of the term sets.

3.3.2 Calibrate Membership Functions

$$\begin{gathered} LV_{RF,f} = \{\mu_{RF,f,1}, \mu_{RF,f,2}, \dots, \mu_{RF,f,p}\}\ \forall\ f \in RF \\ LV_{KRI,k} = \{\mu_{KRI,k,1}, \mu_{KRI,k,2}, \dots, \mu_{KRI,k,p}\}\ \forall\ k \in KRI \end{gathered} \tag{23}$$

where $\mu_{RF,f,l}$ and $\mu_{KRI,k,l}$ are the l^{th} membership functions in their respective term sets, $l \in \{1, 2, \dots, p\}$.

It is next necessary to attune the KRIs and their consequent risk factors (which are, respectively, the model's independent and dependent variables) with their fuzzy representations. In other words, the calibration process is one in which crisp values of the input variables are associated with that universe's descriptive subsets, effectively converting the numerical input to a linguistic one. Membership functions encode this association as a degree of truth to which the associated variable is considered a member of the linguistic set. In this sense, articulation of the model's parameters "requires a shift in knowledge representation from logical determinism and arithmetic formalism to a semantics and property-based representation [that is] expressed directly through the surface characteristics of fuzzy sets" (Cox, 492). The process of calibrating the membership functions determines the important fuzzy set characteristics of shape and overlap, the latter of which is tantamount to set ambiguity, or fuzziness.

While Chapter II's discussion of membership function selection gives a comprehensive accounting of a concept's characteristic shape, the *horizontal method* is employed as an experimental approach for apprising a function's construct in the proposed FIS methodology for several advantageous considerations. In particular, conduct of the sampling largely coincides with existing operational planning processes; staff sections are regarded as the expert population corresponding to the subject matter for which they are responsible. This is not inconsistent with current practice in which the staff identifies and elevates risks within their functional area, albeit chiefly in qualitative fashion. Where quantitative measures are presently introduced, however, there exists a distinct lack of standardization that breeds divergent consistency in evaluation; the resulting risk assessments are ultimately incommensurable and not well-suited to comprehensive course of action comparison. Not only does the *horizontal method* address this need for standardization, but even in its experimental simplicity it is capable of delivering "reliable and significant estimates" (Pedrycz, 19).

Essentially, the method consists of surveying n number of experts as to whether a given sample value, x_{ks}, is compatible with the term l in universe of discourse $\boldsymbol{X}_k$. This is posed as a question accepting only binary responses in the positive $P_i(x_{ks})$ or negative $N_i(x_{ks})$, from which the estimated degree of truth at value x_k is the ratio of positive to total responses such that

$$\mu_{KRI,k,l}(x_{ks}) = \frac{\sum_i w_i P_i(x_{ks})}{\sum_i w_i P_i(x_{ks}) + w_i N_i(x_{ks})} \tag{24}$$

where $i \in \{1, 2, 3 \ldots, n\}$ is the index of the responding expert, w_i is an optional weight attributing some level of authority or expertise to expert i, and the positive and negative variables are mutually exclusive indicators, $P_i(x_k) = 1 - N_i(x_k) \in \{0,1\}$. The survey is conducted over some selected number of elements x_{ks} of universe $\boldsymbol{X}_k$, where s signifies the distinct element sampled. The set of truth values so determined not only serves as the basis for fitting a distribution and, accordingly, the fuzzy set l's membership function, but in fact defines the fuzzy set's bounds via the result's standard deviation,

$$\left[\mu_{KRI,k,l}(x_{ks}) - \sqrt{\frac{\mu_{KRI,k,l}(x_{ks})\left(1 - \mu_{KRI,k,l}(x_{ks})\right)}{\sum_i w_i P_i(x_{ks}) + w_i N_i(x_{ks})}}, \; \mu_{KRI,k,l}(x_{ks}) + \sqrt{\frac{\mu_{KRI,k,l}(x_{ks})\left(1 - \mu_{KRI,k,l}(x_{ks})\right)}{\sum_i w_i P_i(x_{ks}) + w_i N_i(x_{ks})}}\right] \tag{25}$$

To ensure commensurability of the output fuzzy sets, thereby permitting the combination of risk factor conclusions into a single operational risk value, it is necessary that the analyst define risk factors over the same universe of discourse, $\boldsymbol{Y}_f$. This does not, however, require that each risk factor share the same number or shape of subordinate linguistic descriptions, and each membership function, $\mu_{RF,f,l}(y_f)$, can be defined according to its conditional interpretation. However, that the RAFIS will, in fact, permit alternative definitions of the consequent universes. Should risk decisions demand more concrete considerations than an abstract risk value, it is possible that model outputs be instituted in the language of "mission success (which missions will and which will not be accomplished), time (how much longer will a mission take to achieve success), and forces (casualties, future readiness, etc.), and [to a lesser extent] political implications" (JP-5, V-14). Regardless, the outcome of this process is that all term sets, $LV_{RF,f,l}$ and $LV_{KRI,k,l}$, are defined with each elemental membership function fully parameterized.

Continuing with the example problem introduced in Section 3.3.1, the fuzzy sets are depicted in Figure 18; for the purpose of illustration, the assumed membership functions of the *Probability* KRI are Gaussian, *Severity*'s are triangular, and those of the Risk Factor, *Risk Level*, are trapezoidal. These membership function shapes are selected to delineate the behaviors from one another in later exposition; they are not, however, chosen arbitrarily but are rather representative of the ambiguity intrinsic to the linguistic concepts. Note also that the process of eliciting the characteristics of the membership functions simultaneously aids in defining the universe of discourse's domain (such that the minimum value of the qualitatively lowest fuzzy set and the maximum value of the qualitatively highest define the universal boundaries).

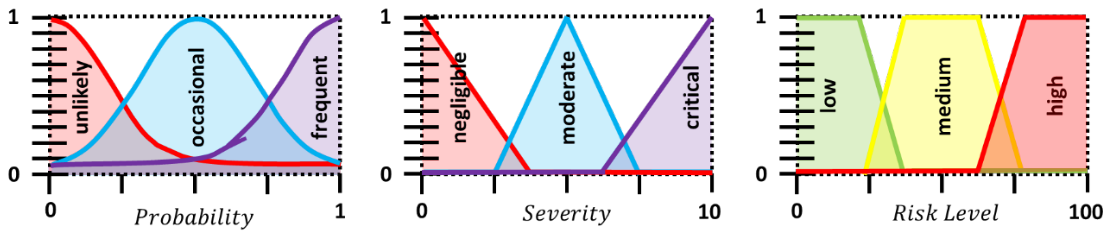


Figure 18. Example Antecedent and Consequent Membership Functions.

3.3.3 Specify Inference Rules

$$R_1 = \text{ If } (\boldsymbol{KRI_1} \text{ is } \mu_{KRI,1,1}) \text{ and } (\boldsymbol{KRI_2} \text{ is } \mu_{KRI,2,1}) \text{ then } (\boldsymbol{RF_1} \text{ is } \mu_{RF,1,1}),$$

$$R_2 = \text{ If } (\boldsymbol{KRI_1} \text{ is } \mu_{KRI,1,2}) \text{ and } (\boldsymbol{KRI_2} \text{ is } \mu_{KRI,2,2}) \text{ then } (\boldsymbol{RF_1} \text{ is } \mu_{RF,1,2}), \quad (26)$$

$$R_j = \text{ If } (\boldsymbol{KRI_k} \text{ is } \mu_{KRI,k,l}) \text{ and } (\boldsymbol{KRI_{k'}} \text{ is } \mu_{KRI,k',l}) \text{ then } (\boldsymbol{RF_f} \text{ is } \mu_{RF,f,l}),$$

and so forth…

where R_j is the j^{th} inference rule.

In this third compositional step of knowledge elicitation, expert knowledge is deconstructed and encoded into the system through development of a rule base that contains a series of *inference rules* using natural language to specify the interactions between KRIs and their causal relationships with the consequent Risk Factors, thereby “mimicking human’s reasoning capabilities to solve complex systems” (Reveiz, 24). The rule base therefore describes the expected behavior of the output Risk Factors given the input KRI variables (and, in turn, the degree to which the KRI’s term sets are satisfied). The rules, though linguistic, constitute the model’s mathematical processes; they encode the fundamental transformations executed on the input data and ultimately give shape to a response surface that reacts in a predictable and contextually appropriate manner, yielding the model's output data. Construction of the model's rule base is, in terms of the JRAM, predominantly captured by *Risk Assessment* in that it formalizes the linkages between the hazards and their potential consequences (in consideration of the risk drivers). It is the actual execution of the model, however, results in the *Risk Characterization* answering "how much risk?" (JRAM, B-3).

There exist no hard and fast procedures for the development of the rule base. In the previous step, the vocabulary of the model's fuzzy sets is used to define the semantic properties that underlie the relationship between the functional area expert and the decision process. In this step, the knowledge engineer's task is to codify the expert's decision and judgement protocols in a series of inference rules that manipulate those fuzzy sets according to the fuzzy operators used in the propositional calculus (Cox, 493). The structure of the inference rules is discussed in Chapter II's introduction to "fuzzy reasoning." In general practice, however, the rule base should cover every possible combination of antecedent linguistic variable that contributes to the formation of some consequent space. Consider the first three rules of this Chapter's ongoing example:

R_1 = If (**Probability** is *Unlikely*) and (**Severity** is *Negligible*)
then (**Risk** is *Low*)

R_2 = If (**Probability** is *Occasional*) or (**Severity** is *Moderate*)
then (**Risk** is *Medium*) (27)

R_3 = If (**Probability** is *Frequent*) or (**Severity** is *Critical*)
then (**Risk** is *High*)

While not exhaustively listing every possible combination of antecedents, the rules illustrate how the subject matter expert might exercise judgment in construction of the logical rule base. Of course, the disjunctive case (of "or") is clearly irregular or even inappropriate for the ruleset governing the interaction of *Probability* and *Severity*; expected risk is often calculated as the expectation (product) of these two factors. Use of the conjunction "and" would permit consistency within the ruleset and allow independent definition of all possible pairwise combinations. This example is constructed to depict the use of both logical connectives for the sake of exposition; inconsistency with the defined rule base, specifically resulting in non-monotonicity of the output surface, is acknowledged.

Probability \ *Severity*	*Neg*	*Mod*	*Crit*
Frq	**M**	**H**	**H**
Occ	**L**	**M**	**M**
Unl	**L**	**L**	**L**

Figure 19. Example Fuzzy Associative Memory (FAM).

Nevertheless, a convenient and commonly used notation for recording the rule base is the Fuzzy Associative Memory, or FAM, which captures in tabular form the transformations of fuzzy sets to others (Kosko, 306). While the example's dual-input, single-output formulation is intuitively recorded in a compact representation, complex risk decisions may, in terms of the RAFIS, require m number of n-dimensional hypercubes, each dimension quantized by p. The rule base is the functional mechanic of the model itself; the clarity and comprehension of rules are vital to the model's "maintainability, quality, and expandability" (Cox, 554). The example's FAM in Figure 19 is derived from an extension of the previously listed rules; it was purposely constructed to resemble a traditional risk matrix (particularly considering the axes of likelihood and consequence). While the divorce of fuzzy logic from probability theory bears repeating, and is not a required component in the model, it is not prohibited from inclusion either, and the example is in fact a viable implementation of the RAFIS. The example's risk matrix will later be compared with its fuzzy counterpart.

3.4 Implementation of the RAFIS and Selection of Fuzzy Operators

The topology of the Risk Factors' consequent space is contingent on the mathematical operators applied during the logical processes of composition, implication, aggregation, and defuzzification. While the rest of this chapter addresses each of these processes in sequence, several basic *t-norms* and *t-conorms* are introduced in Chapter II that may serve as the fuzzy operators used to perform the transformations at each stage of the model. The RAFIS employs a Mamdani-type fuzzy inference system, introduced in 1975 as one of the first applications of a rule-based fuzzy logic controller and widely considered the most commonly adopted inference technique. The Mamdani fuzzy logic controller is characterized by membership functions that define the consequent spaces as fuzzy sets, therefore requiring a disjunctive aggregation of rules and necessitating a process of defuzzification to interpolate a crisp value for each output variable (Yuan, 835). Mamdani-types also utilize the supremum-minimum (*sup-min*) compositional rule of inference for approximate reasoning postulated by Zadeh (1975, 28). Finally, Castro (1995) proved that the Mamdani class of controllers are universal approximators; that is, as a fuzzy logic it is capable of approximating any real continuous function defined on a compact domain to any arbitrary accuracy. This contribution provides the theoretical foundation for their application over and above qualitative justifications that exploit the inherent nature of their linguistic reasoning; in effect, it addresses the question of why fuzzy rule-based systems "have such good performance for a wide variety of practical problems" (Castro, 629). For these reasons, the RAFIS' Fuzzy Inference Engine utilizes the *min* and *max* operators for fuzzy conjunction and disjunction in composition, the *min* operator in implication, the *sup* operator for aggregation, and the *centroid* method of defuzzification.

3.4.1 Fuzzification

$$\mu_{KRI,k,l}(x_k), k \in KRI, l \in R_j, j \in R \; \forall \; x_k \in \boldsymbol{x} \quad (28)$$

Given the operational risk factors and their KRIs determined in the knowledge elicitation process, the associated input variables, x_k, are first measured or forecast per KRI, per risk. In the first step of the RAFIS' logical process, known as *fuzzification*, the crisp (non-fuzzy numerical) values are mapped to the corresponding fuzzy (linguistic) sets as prescribed for that variable's universe of discourse (clearly, any variable assessed outside the universe's boundaries is problematic; barring extreme "Black Swan" events in which the paradigm must necessarily be readjusted, such a situation is indicative of an inadequately constructed universe of discourse and fuzzy set parameters) (Taleb, xxii). The mapping is accomplished by way of evaluating each membership function given the input metric, per element of each propositional antecedent, per logical rule. For each evaluated membership function, the output of this first step is the degree of truth to which the input's measurement is considered to belong to the fuzzy set, effectively "fuzzifying" the formerly crisp input.

Continuing with the chapter's example, fuzzification occurs in each instance where an input metric is mapped to a fuzzy set associated with an element of a rule's antecedent. Figure 20 demonstrates this application to the first premise of the second rule, (**Probability** is *Occasional*). This can be interpreted to mean that a value of $x = 0.7$ in universe of discourse $\boldsymbol{X}$ is compatible with the linguistic concept of "occasional" probability to a degree of 0.642; equivalently, the extent to which the probability is considered occasional. All inputs are likewise fuzzified over each elemental premise for each rule.

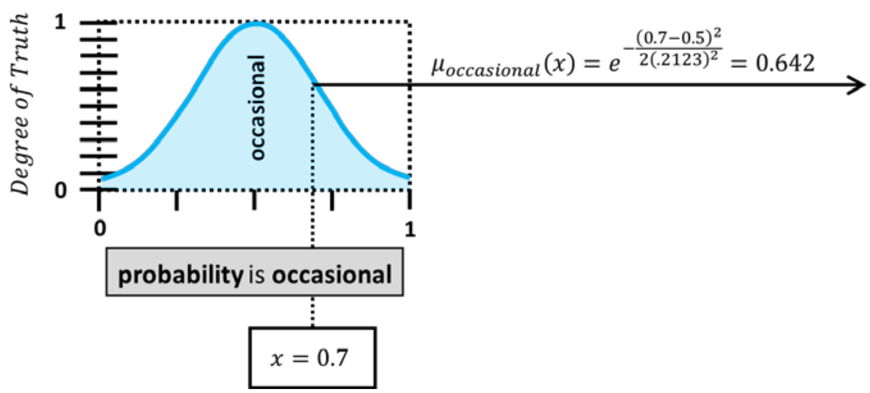


Figure 20. Example Fuzzification of Input Variable.
Source: Author's elaboration based on Mathworks (1-29).

3.4.2 Composition

$$\mu_{Aj}(\boldsymbol{x}) = \bigwedge_{k \in KRI, l \in Rj} \mu_{KRI,k,l}(x_k) \ \forall \ j \in R \tag{29}$$

where $\mu_{Aj}(x)$ is the composite (singleton) antecedent truth value for rule j, compounded via conjunction, and evaluated for all rules.

The second component of the RAFIS' Inference Engine, herein termed *composition*, accepts as input the already fuzzified truth degree of each rule's individually evaluated premises, applies (if present; some rules may only contain a single premise) the fuzzy set operator that corresponds with the logical connective between those premises (*and*, *or*, *not*), and ultimately forms a composite antecedent that yields a single fuzzy truth value representative of the degree to which the rule's antecedent is comprehensively satisfied. Composition is conducted in parallel against the whole ruleset. The formulation proposes a grand intersection of the entire antecedent based on the conjunctive structure consistent with the FAM rule base; that is, all predicate expressions are joined by "and." The preceding discussion of universal approximators establishes that any number of mappings meeting the criteria of *t-norms* (for the logical conjunction, and) and *s-norms* (for the logical disjunction, or) are sufficient to perform the mathematical operations of intersection and union, respectively. The most common operators for conjunctions are minimum and ordinary product; for disjunctions, maximum and algebraic sum. While the RAFIS will necessarily accept as final antecedent truth the minimum of their collective truths, the example demonstrates use of the maximum in the disjunctive case.

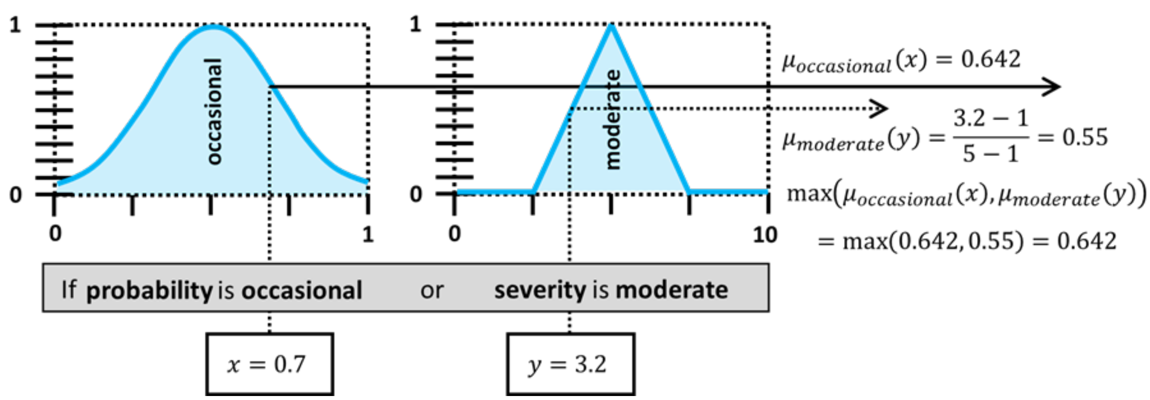


Figure 21. Example Composition of Antecedents.
Source: Author's elaboration based on Mathworks (1-30).

Figure 21 builds on the example by compositing the two antecedent premises of the second rule, (**Probability** is *Occasional*) or (**Severity** is *Moderate*), having already been evaluated at $x = 0.7$ and $y = 3.2$. For expository purposes, the antecedent is a disjunction and dictates use of a *s-norm*; the maximum operator is selected. Of the two degrees of truth for occasional probability and moderate severity, 0.642 and 0.55 respectively, the former achieves the maximum, and is the output value of this step. Other rules in the formulation are evaluated likewise, in parallel.

3.4.3 Implication

$$\mu_{C'_j}(y_f) = \mu_{Aj}(\boldsymbol{x}) \wedge \mu_{RF,f,l}(y_f) \; \forall \; f \in RF, j \in R \tag{30}$$

where $\mu_{C'_j}(y_f)$ is the partial consequent truth value of risk factor f for rule j, evaluated for all risk factors in rule j, for all rules.

The third subprocess of the RAFIS is *implication* of the, now composite, antecedent to the consequent, predicated on the specified rule. For each rule evaluated in parallel, implication involves the input of a single, fuzzified, independent variable and, as an output, modifies the fuzzy set associated with that rule's consequent by the degree of the input variable. More clearly, the antecedent's composite value specifies the level of support that the proposition's consequent is true. This is accomplished in one of two ways. In the first, the minimum operator truncates the consequent's fuzzy set; utilizing the representation theorem, this method effectively discounts the support region of the fuzzy set's α-cut where α is equal to the antecedent's composite value. Alternatively, the ordinary product operator may be used as a scalar of the consequent's fuzzy set. It is also important to note that a weighted ruleset may be optionally implemented to mitigate the importance of individual rules relative to the others; weights in the interval of $[0,1]$ are applied to the antecedent's output prior to implication and are otherwise assumed to be 1 (Mathworks, 1-31). Figure

22 depicts the truncation achieved by the minimum operator on the (**Risk** is *Medium*) consequent set; specific to the example, this is

$$Supp(med_{\alpha=0}) - Supp(med_{\alpha=0.642}) = \{z \in med | \mu_{med}(z) < 0.642\}. \quad (31)$$

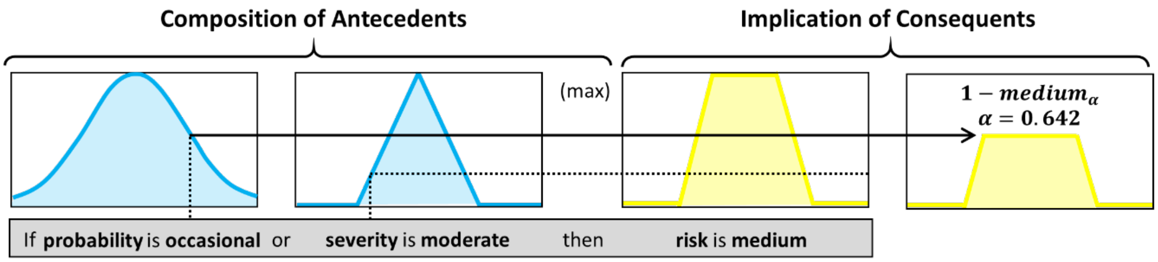


Figure 22. Example Implication of Consequents.
Source: Author's elaboration based on Mathworks (1-31).

3.4.4 Aggregation

$$\mu_C(y_f) = \bigvee_{j \in R} \mu_{C'_j}(y_f) \; \forall f \in RF \quad (32)$$

where $\mu_C(y_f)$ is the consequent truth value for risk factor f aggregated over the entire rule base, for all risk factors f.

Accepting as input the truncated or scaled fuzzy set outputs of the implication subprocess, the fourth component of the RAFIS aggregates, or compiles, the residual consequents into a single fuzzy set. This output set of the *aggregation*, illustrated in the right-most column of Figure 23, is representative of the support for the consequent given the propositional set of all non-fuzzy inputs evaluated over all rules. In fact, the aggregate membership function is specific to the particular crisp inputs; alternate inputs may generate a distinct membership function. Mathematically, the aggregation is executed through any commutative operator for the logical disjunction (such that the ordinal sequence of inclusion is immaterial); the maximum, ordinary sum (over all values of $\boldsymbol{Z}$), and algebraic sum ($probor$) are all used in the literature as *t-conorms* to determine multi-set union. In the example, aggregation via maximum of the three resultant consequent fuzzy sets yields the membership function shown in the bottom-right panel of Figure 23. In a broad sense, the aggregate membership function is therefore

$$\begin{aligned}(\mu_{low \cup med \cup high})(z) \\ = \max(\mu_{low}(z) < 0.004, \mu_{med}(z) < 0.642, \mu_{high}(z) < 0.368).\end{aligned} \tag{33}$$

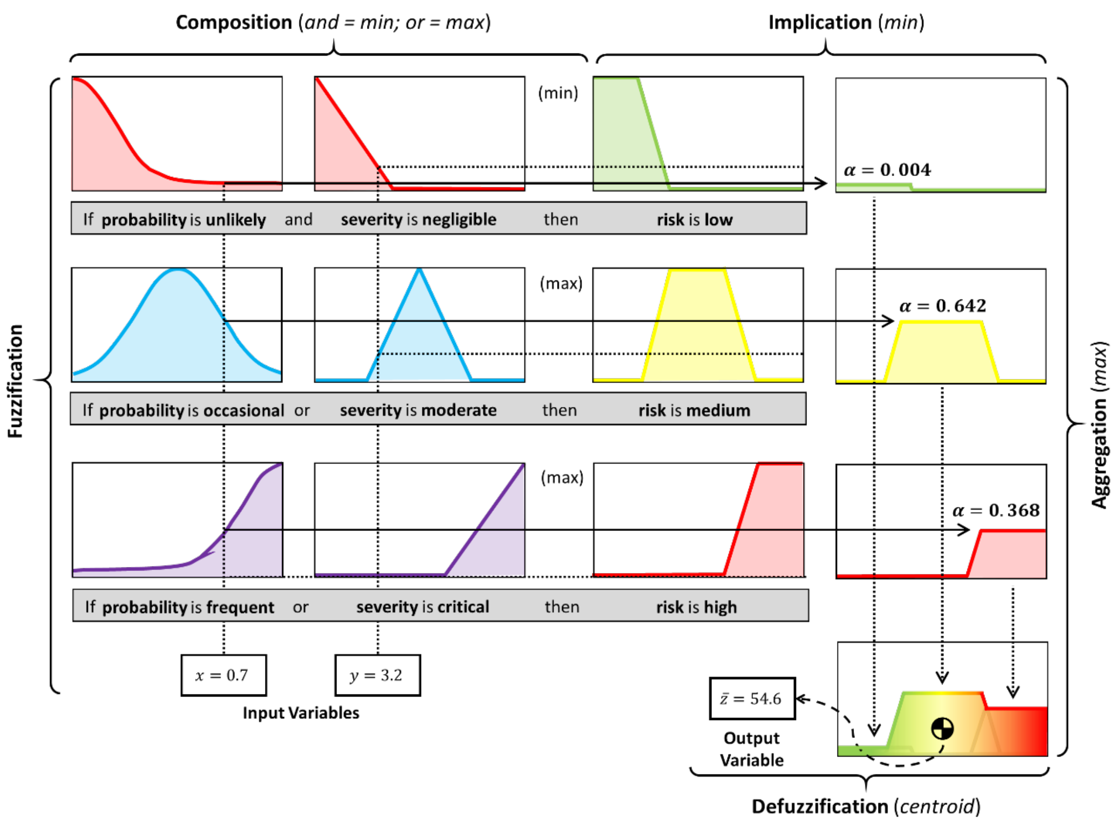


Figure 23. Example Inference Detail.
Source: Author's elaboration based on Mathworks (1-36).

3.4.5 Defuzzification

$$\bar{y}_f = \frac{\int_{\check{y}_f}^{\hat{y}_f} y_f \, \mu_C(y_f) \, dy_f}{\int_{\check{y}_f}^{\hat{y}_f} \mu_C(y_f) \, dy_f} \cong \frac{\sum_{i=0}^{n} y_{fi} \mu_C(y_{fi})}{\sum_{i=0}^{n} \mu_C(y_{fi})} \forall f \in RF \tag{34}$$

where $\check{y}_f$ and $\hat{y}_f$ are the minimum and maximum bounds on the universe of discourse Y_f, y_{fi} reflects the discretized points over the domain of Y_f, and $\bar{y}_f$ is the singleton composite moment that represents the fuzzy consequent space of risk factor f.

The fifth and final step of the FIS is *defuzzification*, which is effectively the extrapolation of a crisp value that is representative of the aggregate fuzzy set. This single value is the JRAM's *Risk Characterization*, and answers the question of "how much risk?" As stated in the prior paragraph's explanation of aggregation, the aggregate fuzzy membership function is indicative of the degree of truth to which the consequent is satisfied for all values of z in $\boldsymbol{Z}$ (given the specific input values, it is likely that all values of z correspond with some nonzero truth degree). Accordingly, it is useful to extract a non-fuzzy value, $\bar{z}$, as output from the FIS from which further quantitative analysis can be conducted. While textbooks like Cox (1999) provide a more comprehensive accounting of the various methods employed for defuzzification (centroid, bisector, mean of maxima), the centroid calculation is predominantly practiced. In this method, also known as the "center of gravity" method, an expected value is determined by weighted average of the area under the aggregate membership function's curve. This value corresponds to the point in the z-axis which equally bifurcates the output fuzzy region by area (Reveiz, 15). Applied to the example in Figure 23, centroid defuzzification yields the FIS output of $\bar{z} = 54.6$ (out of 100), indicating the level of risk that corresponds with the specified input variables. On its own, this value can be evaluated against the consequent membership functions for Risk Level. In this case, the value coincides with medium risk. However, the output value is more informative when considered in relation to evaluations utilizing alternate inputs variables, as would be the case in Course of Action comparisons of the JPP.

3.4.6 Risk Factor Consolidation

While the output aggregate risk factors, $\bar{y}_f$, may simply be summed (or weighted and summed) to determine a total operational risk value, it is probable that dependencies exist between the individual risk factors that inhibit use of a strictly summative method for consolidation. In such cases, a correlation matrix can be used to account for the dependent relationships between output variables. The correlations may be derived from either experience data or expert opinion. The model's total operational risk is therefore defined by

$$OR = \sqrt{(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_m)\begin{pmatrix} 1 & \rho_{12} & \rho_{1m} \\ \rho_{21} & 1 & \rho_{2m} \\ \rho_{m1} & \rho_{m2} & 1 \end{pmatrix}\begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_m \end{pmatrix}} \qquad (35)$$

where OR is the total operational risk, ρ_{ij} is the correlation coefficient of risk factors i and j, and m is the total number of risk factors (Shang, 36).

3.5 Inform Decision-Making

Having fully assembled a knowledge base consisting of membership functions and their corresponding propositional rule base, and having instituted the mathematical framework of triangular norm mappings particular to each of the model's subprocesses, the RAFIS effectively defines the feasible region of the consequent space for all possible combinations of input variables. Accordingly, the relationship of the consequent to any two antecedent variables may be modeled as a surface. In this manner, the pairwise examination of two inputs and their effect on the resultant output surface may be used to both validate the model's encoded expertise (by way of membership functions and inference rules) but also inform decision-makers of the true non-linear nature of the relationship under consideration. While any arbitrary risk ontology is certainly likely to consist of more than two KRIs, the example, consisting exclusively of probability and severity, generates a risk surface in that is akin to the traditional risk matrix in common use, but tailored to the problem and without susceptibility to the complications enumerated in Chapter II. Figure 24's risk surface, or 'fuzzy risk matrix,' intuitively increases as both probability and severity increase; in this case, it is evident from the gradient associated with severity that the risk accrues more rapidly as a result of an increase in this factor, relative to probability. If attempting to mitigate overall risk through application of limited resources, this insight may indicate a better return of investment by addressing severity over probability. An alternate perspective may wish to pursue system stability or robustness by targeting flat or gradual surface regions not adjacent to precision-sensitive high gradient slopes.

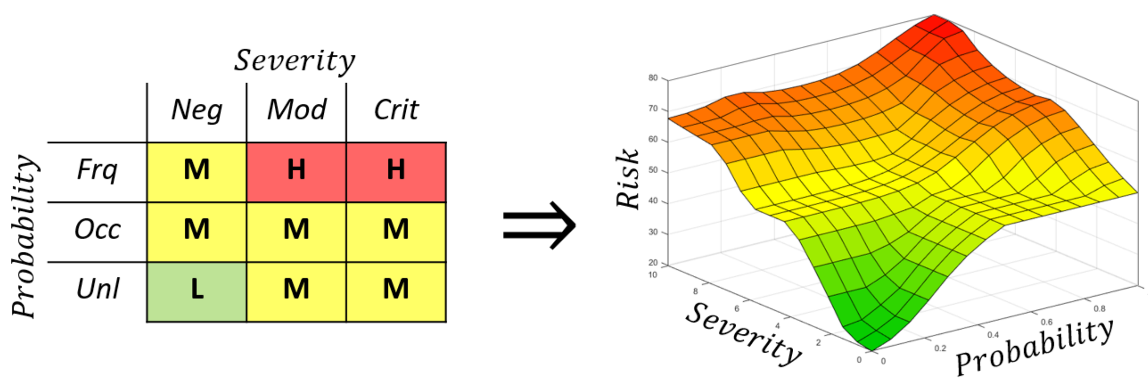


Figure 24. Example Risk Surface (Fuzzy Risk Matrix).

While the individual Risk Factor output variables, the consolidated Operational Risk value, and the visual depictions of risk through fuzzy risk matrices constitute the products of the RAFIS, the end result is that these products aid the decision-maker in the JRAM's *Risk Evaluation,* answering "how much risk is OK?" The RAFIS uniquely provides a numerical evaluation of risk that would otherwise have been a qualitative exercise. It also provides an expedient and consistent means for the reevaluation of risk following the implementation of *Risk Management*'s mitigation efforts, and, through the integrative and informative processes of staff estimates and operation assessment, may be updated and utilized throughout operational execution as a supplementary process to assessment.

IV. Analysis

"An expert is someone who knows some of the worst mistakes that can be made in the concerned subject, and therefore understands how to avoid them."[10]

- Werner Heisenberg
Der Teil und das Ganze (1969, 281)

4.1 Chapter Overview

This chapter demonstrates the use of the RAFIS through an abbreviated instance of a notional rotary wing tactical mission. The purpose is to examine the feasibility of building and evaluating a moderately-sized instantiation of the model, as well as considering its potential applicability to the problem of quantifying operational risk in general. Section 4.2 discusses the model's construction; it is principally concerned with Knowledge Elicitation and Problem Framing. The focus of Section 4.3 is Risk Assessment and Characterization. The Joint Planning Process is oriented on planning activities at the strategic and operational level; however, this tactical mission is examined due to the prevalence and accessibility of risk assessment documentation and tools employed in the management of aviation risk. Military aviation breeds, by its very nature, a risk-conscious and literature-prolific community. This documentation, coupled with the author's experience as a Senior Army Aviator, serves as the knowledge base for the model's construction. It is not the intent of this study to suggest the inadequacy of present aviation risk assessment tools, only to demonstrate use of the proposed model.

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[10] Heisenberg, Werner. (1969). *Der Teil und das Ganze: Gespräche im Umkreis der Atomphysik*. München: R. Piper & Co Verlag. English translation from the original German by the author. Heisenberg is paraphrasing Niels Bohr's recounting of a debate with Philipp Frank on causality and uncertainty during the Copenhagen Congress (The Second International Congress for the Unity of Science; June 21-26, 1936).

4.2 Model Construction

The primary sources of information for the model's knowledge elicitation process are several variants of the 'Risk Management Worksheet', 'Risk Common Operating Picture' (RCOP), and the 'Electronic Risk Assessment Worksheet' (ERAW) employed by Army rotary wing units over the past decade. While the actual mathematical models employed are suspect from a risk-theoretic sense (they are strictly additive, but with conditional considerations), it is possible to elicit from them several knowledge elements vital to construction of the RAFIS. First, they establish a taxonomy of risk activities and risk factor areas from which further conversation may take place. This consideration informs the decision to use a "5 M" contextual lens: *Man*, *Machine*, *Medium* (environment), *Management*, and *Mission*; these will serve as the model's output Risk Factors. Secondly, commonalities between the worksheets reveal an accepted vocabulary, not singularly in linguistic terms, but also semantics; for instance, crew member experience is categorized based on total flight hours, but the practiced categories are not based any regulatory distinction. The collection of language is used to define the risks contributing to the respective Risk Factors, but also the (potentially fuzzy) term sets that constitute the membership function conventions.

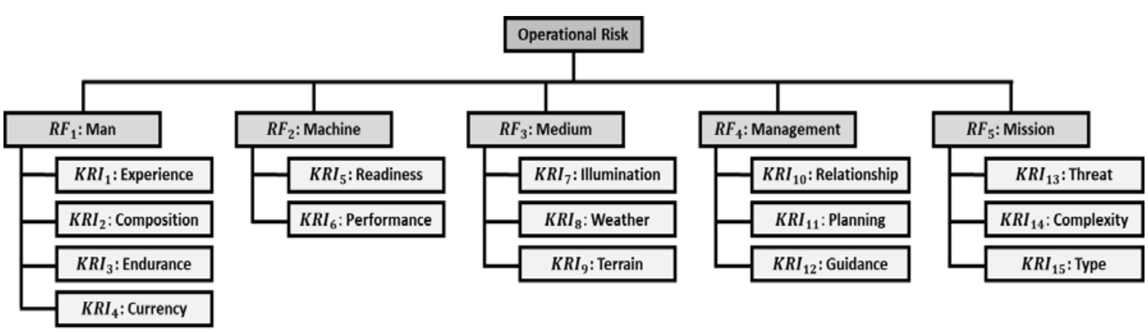

Figure 25. "5 M" Risk Hierarchy.

Figure 25 is the product of this process and decomposes the Operational Risk into five output Risk Factors and 15 input Key Risk Indicators. The same source material provides indication of ranges (universes of discourse) and numerical categories that comprise the KRIs. For example, the *Medium* (environmental) consideration of *Weather* (KRI_8) is designed to account for a comprehensive number of causal phenomena; cloud ceiling, visibility, temperature, pressure altitude, winds, and precipitation are among these. Because of the high correlation of many of these phenomena, expert specification of the associated state variable's value (for *Weather*) is necessary when resolving the model. Were the model to be decomposed further into independent variables, it would be possible to define the ceiling over a range from surface to Flight Level 180, or 18,000 feet (since operation in Class A airspace is necessarily conducted under Instrument Flight Rules; for practical purposes, ceiling might be considered irrelevant above this figure). The functional concern for tactical rotary aircraft is much lower in altitude, however, and there exist some commonly delineated categories at elevations below 500 feet Above Ground Level (AGL), above 500 AGL, above 700 AGL, above 1000 AGL, and above 1500 AGL. These correspond with decreasing degrees of risk.

This discussion does not mean to be an exhaustive dialogue on weather or exclusionary to non-aviators or weathermen. Rather, it emphasizes the expert consultation required as input to formulate each membership function and KRI. Ceilings also serve to illustrate a reason as to why fuzzy thinking is arguably suitable given the already agreed upon crisply-delineated standards in practice. Cloud ceilings, as a weather phenomenon, are not temporally static; they constantly move due to winds. They might be tens of feet or many thousands of feet thick, without distinction. They are not of uniform density or distribution; only broken or overcast conditions are considered to constitute a ceiling and a "measurement" of under 5/8^{ths} of the sky results in a condition of 'no ceiling' (but perhaps infinitesimally shy of that criteria). Likewise, the different tools (ceilometers) used to measure cloud base (ceiling height) are subject to any number of different resolutions, capabilities, or calibrations. There are even mathematical formulations from which it can be calculated absent observation. Needless to say, a tactical mission that executes under assumption of one ceiling condition might actually encounter a very different scenario. Fuzzy logic minimizes the impact of natural variation or faulty forecast by modeling the surface as a continuous function, without having to account for every facet of reality or its measurement.

In this particular implementation, the 15 KRIs are composed of 73 membership functions, each assiduously considered in the same fashion as the discussion on ceilings, and each possessing a unique characteristic shape, parameterization, and degree of fuzzy overlap. Finally, the logical rule base is built in which encodes the qualitative expert judgment on variable interactions. All feasible interactions need be considered; the appropriate number of interactions for this model requires 264 distinct risk judgements, recorded as a set of inductive rules, with each rule individually evaluated by the author. Appendix A contains the entire listing of membership functions, their parameters, and all logical rules. The formulation is then programmed into MATLAB. It is evident that the construction of large instances are demanding affairs; this model is relatively conservative with only 15 inputs and 5 outputs.

4.3 Model Evaluation and Results

A fictional state vector, x, constituting the expected or observed numerical measures corresponding to the scenario's KRIs in Figure 25, is generated as input to test the functionality of the model. The state vector is

$$x = (x_1, x_2, x_k, \dots, x_n) \\ = (900, 2, 18, 26, 83, 13, 19, 720, 6, 1, 50, 2.5, 12, 6, 4), \tag{36}$$

where, by the formulation in Chapter III, x_k is the quantified value corresponding to the k^{th} KRI. For instance, the input associated with KRI_8 is $x_8 = 720$. The other constituent values are not arbitrarily chosen (although it would not be inadmissible by the model if they were); they represent a typical and realistic scenario reflecting appropriate crew selection for the particular mission and conditions. This is intended to convey interpretable results (non-realistic inputs are potentially interpretable, but produce either trivial or absurd results). Within the context of operational planning, these values would be derived from running estimates, wargame results, expert opinion, or actual measurement (enemy threat, for example, might be determined under the Intelligence Collection Plan).

The MATLAB model resolves a single run near-instantaneously on a Windows 10 (x64) based system with dual E5-2680 CPUs at 2.50 GHz and 192 GB of system memory; the parallel nature of rule inference is suggestive of a low computational cost that grants risk analysts the ability to responsively apprise the model for updated state variable input or rapidly assess alternatives. In this regard, and despite the initial rigor of model creation, it is also notable that once formulated, adjusting model parameters is an equally casual process. While changing membership function shapes and individual inference rules are relatively non-invasive, alterations to linguistic lexicon, like the addition or removal of a membership function altogether, demands the analyst's strict attention as there are implications on the linguistic calculus of the rulebase.

Model output comes as crisp numerical values of degree equivalent to the number of Risk Factors. The response vector, $\bar{y}$, reflects the quantitative risk within the individual Risk Factor domains. The response vector corresponding to input vector x is

$$\bar{y} = (\bar{y}_1, \bar{y}_2, \bar{y}_f, \dots, \bar{y}_m) = (4.09, 3.41, 3.6, 5.85, 5.86), \tag{37}$$

where $\bar{y}_f$ is the aggregate and defuzzified value corresponding to the f^{th} Risk Factor as depicted in Figure 25. It is clear that the fifth entry, *Mission Risk*, is the highest Risk Factor with a value of $\bar{y}_5 = 5.86$ (though, correlations, if any, should be taken into account, as they are in Chapter III's method for Risk Factor consolidation). In a vacuum, these output values have meaning only in their relative comparison. Individually, or as an aggregate OR value, they might be used to compare alternative scenarios run under the same model. As a framework for informing risk decisions, however, the fuzzy risk matrices reflect the entire topology of the solution space relative to the chosen pairwise KRIs. Whereas the point estimates are useful for comparing alternatives, decisions that are considerate of the point estimate's adjacent topology are potentially more robust. Figure 26 depicts the three fuzzy risk matrices for *Mission Risk*, since it is the most severe category. There are three possible pairwise comparisons between the KRIs of *Threat*, Complexity, and *Mission Type*.

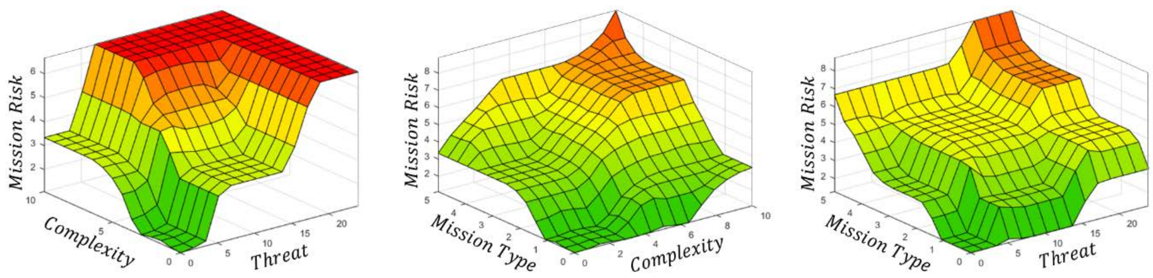

Figure 26. Mission Fuzzy Risk Matrices.

In observing the three Fuzzy Risk Matrices, the objective is to determine a viable approach for the mitigation of *Mission Risk*. For reference, the three input KRI values are 12 for *Threat*, 6 for *Complexity*, and 4 for *Mission Type.* While the digital model allows 3-Dimensional manipulation and rotation of the matrices, printed versions require closer inspection for interpretation and sound diagnoses. A useful technique is to model a cross-sectional profile-view of the respective surfaces, achieved by evaluating the model over the defined range of each KRI while holding all other inputs constant. This is effectively a sensitivity analysis on the variable; it isolates any variability induced by interaction of terms.

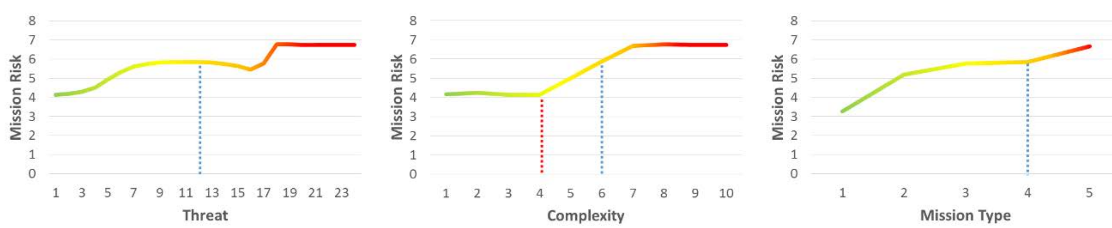

Figure 27. Mission Risk Matrix Cross-Sections.

The cross-sections in Figure 27 correspond with Figure 26's risk matrices for their respective variables. The vertical blue lines reflect the variable's initial (current) value. Supposing that risk reduction for any of the factors demands a resource expenditure commensurate with the degree of deviation desired, then efficient mitigation is achieved by following ‘steepest descent’ paths of least resistance (clearly, this is not tautologically true; it is highly dependent on the resource cost function). *Complexity*’s profile appears most advantageous in this regard. Furthermore, *Threat* and *Mission Type* are, by definition, arguably exogenous to a greater extent than *Complexity*. Dependent on the mission’s criticality, *Mission* may even be an immutable feature of the scenario. Regardless, these are semantic considerations specific to the problem, not the model, and so standard convention dictates that that mission *Complexity* be reduced to the base of the amber-colored slope with a target value of $x_{14} = 4$, indicated by the red line. Achieving this risk mitigation goal reduces the outcome risk vector to

$$\bar{y}^* = (\bar{y}_1, \bar{y}_2, \bar{y}_f, \dots, \bar{y}_m) = (4.09, 3.41, 3.6, 5.85, 4.14), \quad (38)$$

which correctly reflects a change in the single KRI; in practice, the combined response of simultaneous change to multiple KRIs should be examined. Returning to the fuzzy risk matrices, the *Complexity–Threat* interaction reinforces the decision to mitigate *Complexity*, but suggests a reduction in *Threat* only if it can be reduced to a value below $x_{13} = 6$, an unlikely prospect in the scenario. The second figure, *Mission Type-Complexity*, suggests exclusive pursuit of *Complexity* reduction. Lastly, the coordinates of the *Mission Type-Threat* interaction indicates a stable plateau at current levels, and should not be perturbed.

Finally, a close inspection of the three risk matrices and their cross-sections is indicative of some minor erraticism. Specifically, *Threat*’s profile should exhibit monotonic behavior but has a trough at approximately $x_{13} = 17$ which is also observable as a divot in the *Complexity–Threat* surface at about $x_{13} = 17, x_{14} = 6$. In this case, scrutiny of the *Threat* membership functions reveals the cause; underpowered degree of truths are attributed to numerical *Threat* values in the fuzzy intersection of that variable's 'moderate' and 'high' sets, a problem easily resolved by parameter adjustment to increase the amount of overlap. Other potential deficiencies may be the result of an errantly prescribed risk judgement encoded into the rulebase or some unrecognized interaction of terms. The ability to visually inspect response surfaces eases the difficulty of model validation and consistency checking, especially when soliciting feedback from subject matter experts. In the context of the JRAM and RAFIS, the cyclical interaction with staff provides a natural forum for the risk manager to facilitate model refinement. However, fuzzy risk matrices developed for KRI interactions that are designed with non-monotonicity, inverse relationships (negative correspondence), or are purposely and severely non-linear may be unintuitive or of little value for visual interpretation, even if semantically correct in the model's context.

V. Conclusions and Recommendations

"Finally, the general unreliability of all information presents a special problem in war: all action takes places, so to speak, in a kind of twilight, which, like fog or moonlight, often tends to make things seem grotesque and larger than they really are."

- Carl Von Clausewitz
On War (1832, 140)

5.1 Chapter Overview

This chapter summarizes the results of the research with particular emphasis on Sections 5.2.1 through 5.2.4 which answer, in sequence, the Investigative Questions proposed in Chapter I. It also suggests a potential area for future research in Section 5.3, and offers a brief conclusion in Section 5.4.

5.2 Conclusions of Research

In 1965, Lotfi Zadeh established the fundamental concepts of fuzzy sets as an extension of classical set theory that permits the simultaneous membership of an object to diametrically exclusive sets by way of assigning a "grade of membership," or degree of truth, to said object. Zadeh's concept of fuzzy sets reconciles the real-world classes of objects whose membership criteria is imprecisely defined with the sharply defined criteria of random sets, and is therefore suggested to better align with human rationale and logic, "particularly in the domains of pattern recognition, communication of information, and abstraction" (Zadeh, 1965). A distinguishing feature of fuzzy logic is that its application often permits the adequate modeling of a system in which classical set and probability theories are otherwise insufficient. This is due to several dominant characteristics; that fuzzy models are robust against vague or subjectively measured data, that the inference rules necessary for the manipulation of fuzzy membership functions assist in establishing causal relationships among misunderstood or emerging variables, and that the structure of those membership functions readily accept linguistic variable input. The stated objective of this thesis is to begin the development of a viable method for the quantitative assessment of military operational risk in joint planning. It is natural that military operational risk, encompassed by the very difficulties that fuzzy logic addresses, be modeled using such a methodology.

Four investigative questions (IQ) are proposed in Chapter I in order to structure the direction and content of the research. They are addressed in the succeeding paragraphs.

5.2.1 Investigative Question 1.

"How is operational risk addressed in current joint and Service literature?"

While the Service literatures are antiquated in their treatments on operational risk, the *Joint Risk Analysis* presents a conceptual framework that is both more doctrinally and mathematically sound. In particular, it recognizes the inconsistencies present in the Service literature, driven by discrete categorizations, and instead provides the military risk practitioner with a contour graph that properly acknowledges the interaction of probability and consequence as an ambiguous, but continuous, function. However, it fails to suggest any practical means of determining the exact nature of the interaction, leaving said risk practitioner with only a nebulous and philosophical conception of the framework's meaning or potential use. In fact, the literature writ large is reticent on the issue of quantitative methodologies; only in a few instances does it suggest their utility, but almost as abruptly dismisses them in favor qualitative assessments made in *linguistic* terms. In this sense, each Service possesses a distinct and often conflicting vocabulary that disallows inter-Service commensuration. Nevertheless, the conspicuous reluctance to prescribe a fixed mathematical formulation for risk quantification is not inappropriate; it withholds calculation in recognition of the interplay of fog and friction on the battlefield. In short, military conflict is characterized by incomplete and unreliable knowledge, a disappointing prospect for models reliant on probability and experience data, as are the risk matrices common to the Service literature.

5.2.2 Investigative Question 2.

"What challenges are presented by the current doctrinal means of quantitative risk evaluation?"

Risk matrices, the dominant method for communicating and interpreting risk within the DoD, are, despite their apparent benefits, rife with potential complications that proceed from their basic mathematical premise of expected value, instead provoking fallacy and misinterpretation in execution. Risk matrices are demonstrably incapable of axiomatic application in that they cannot simultaneously satisfy monotonicity and soundness, a condition equating to discriminatory impotence. They additionally suffer from the potential for rank reversal, range compression, and are inherently error prone. The combination of these factors result in poor resolution, uninformative categorization, and the suboptimal allocation of limited resources. In aggregate, these limitations "suggest that risk matrices should be used with caution, and only with careful explanations of embedded judgments" (Cox [2], 497). Doctrine attempts to circumvent this general unreliability by advocating for their use as an aid in the subjective assignment of risk level by subject matter experts, rather than their utility in objective quantification. However, inattentive contextual consideration, especially in instances of negative probability-consequence correlation, may produce outright harmful results in which the ‘cure is worse than the disease.’

5.2.3 Investigative Question 3.

“What are the characteristics of fuzzy logic that suggest its ability to reconcile quantitative risk evaluation with its inherent challenges?”

The approximate reasoning afforded by fuzzy inference systems “facilitates the representation of systems for which no… reasonable mathematical models exist [and for] systems which exhibit very complex and nonlinear behaviors” (Cox, 494). They provide, in effect, what amounts to model-free function estimation in that they resolve continuous functions absent *a priori* knowledge of the mathematical input-output relationships involved. Instead of these mathematical functions, subject matter experts articulate (imprecisely) the set of rules that dictates their behavior and approximates the correspondence between the propositional antecedents and consequents. The semantics of the respective predicates is captured in the linguistic variables of fuzzy sets, thereby associating the input and output spaces of two causally-linked fuzzy concepts. Ultimately, fuzzy systems are more flexible than their probabilistic counterparts; imprecision and ambiguity are characteristic features of a model’s structure rather than a forced element of its outcome. In addition to being universal approximators, Castro (1995, 629) and Cox (1999, 495) suggest that the good performance of fuzzy logic control systems can be attributed to their:

- utilization of linguistic information,

- simulation of human thinking procedure,
- ability to capture the approximate and inexact nature of the real world,
- reduction of contradictory solutions to a fuzzy surface, thereby reflecting imprecision in probabilities,
- provision of intuitively expressing concepts for which probability distributions are unknown,
- granting of mathematically sound and semantic-based modeling capability at a high level of abstraction.

5.2.4 Investigative Question 4.

"Is the proposed model, a fuzzy inference system, suitable for the quantification of risk within the current military planning and risk frameworks?"

To an extent, in that it addresses the dichotomy between qualitative and quantitative methodologies, IQ4 is answered in the affirmative. In broad terms, it resolves the inconsistencies present in the DoD's flawed use of risk matrices, incorporates the ambiguity present in linguistic categorization, and conducts logical reasoning utilizing natural language of expert opinion. It capably manages the input of complex, poorly understood, and vaguely-defined problems. It also generates as a byproduct of inference 'fuzzy risk matrices' which are the response surfaces attributed to the pairwise comparison of two select input variables. When the output space is defined solely by those two contributing variables, then the 'fuzzy risk matrix' is likened to a traditional risk matrix for its intuitive and visually tractable nature; the logic, while concealed in an explicit sense, is generally transparent and facilitates an understandable discussion that promotes meaningful risk decisions. However, the complex interactions observed in Chapter IV's model reveal an unintuitive relationship when several or more inputs contribute to construction of the model's surface.

Incorporated as a subordinate mechanism of the JRAM, and acting in concert with the JPP, the RAFIS is a complementary and integrative process whose inclusion is non-disruptive to present practices, leverages for the purpose of knowledge elicitation the information vectors already existent in planning doctrine, and promotes cross-staff synchronization in consuming risk data and driving the cyclical progression of risk communication and decision-making. However, a turnkey adoption of the methodology is problematic for two prevailing reasons. First, while conceptually intuitive, the construction of an initial model is a time-consuming procedure that requires the careful stewardship of the knowledge elicitation process as well as meticulous attention on the part of the knowledge engineer when specifying model parameters and inference rules. Second, the actual execution necessitates use of a robust program for mathematical calculation. While MATLAB's Fuzzy Logic Toolbox is acceptable for modeling small- to medium-sized formulations, the graphical user interface is insufficient for large problems. Additionally, staffs may not possess the MATLAB licensing.

5.3 Recommendations for Future Research

Eliciting an inferential rule base sufficient for the operation of large scale models is a laborious process at best. Even in medium-sized executions, the ability to maintain consistency and confer meaning in the manual risk judgment of all pairwise combinations of Key Risk Indicators is a challenging prospect. Fuzzy adaptive systems are artificial neural networks that incorporate machine learning via embedded parameter estimators to "adaptively infer and modify [their] fuzzy associations from representative numerical samples" (Kosko, 18). That is, provided training data, an adaptive fuzzy system may generate and refine fuzzy rule sets that not only relieves human operators of tedious replication but ultimately confers improved system performance. Specifically, future research should concentrate on the development of a Takagi-Sugeno type Adaptive Neuro-Fuzzy Inference System (ANFIS) for risk appraisal. The foundational literature is *Derivation of Fuzzy Control Rules from Human Operator's Control Actions* (Takagi & Sugeno, 1983) and *ANFIS: Adaptive-Network-Based Fuzzy Inference System* (Jang, 1993). The extension of the RAFIS to a 'RANFIS' may also have utility in the development of artificial intelligence assisted decision-making capable of leveraging the massive data streams generated from modern battlefields, or even in the emergence of autonomous systems making independent risk valuations while waging algorithmic warfare.

5.4 Summary

As an attempt to begin the development of a methodology for the quantitative assessment of operational risk, the RAFIS shows promise in confronting the known challenges; ambiguity is encoded as linguistic variables (fuzzy sets) whose composition is a continuous surface, rule-based logic addresses entangled causal relationships in a manner that mimics human approximate reasoning, and it produces meaningful crisp outputs and visual representations (fuzzy risk matrices). Most significantly, its real benefit is as an expert system; it recognizes the military imperative of subjective assessment by experts in their respective risk domains, and provides, perhaps, some fleeting insight into the *coup* *d'oeil* that enables their qualitative judgments. The complexities of modern battlefields will only continue to compound, demanding decision support systems that are flexible, robust, and rapid in promoting a common understanding of emerging and dynamically evolving risks. While not a panacea for wholesale quantification of military operational risk, fuzzy logic may offer a credible alternative for informing risk decisions through its inheritance of the conscious, educated, and experienced thought of contributing experts.

Appendix A. MATLAB Fuzzy Inference System Formulation (.fis)

This appendix presents the entire formulation of the Fuzzy Inference System as implemented in Chapter IV. The model was executed using MATLAB's Fuzzy Logic Toolbox and consists of 15 input variables (KRIs), 5 output variables (Risk Factors), and 264 inference rules. While the variables definitions are intuitive, the format of the inference rules is:

$$input(mem\ funct), output(mem\ funct)\ (weight) : (connective),$$

where there are 15 *input* columns, 5 *output* columns, and a *connective* value of (1) indicates the conjunctive case. In observing the rules, it is insightful to consider the sparsity of the array; the rules only assess the KRIs that contribute to the associated Risk Factor.

```
[System]
Name='RAFIS_FINALv1'
Type='mamdani'
Version=2.0
NumInputs=15
NumOutputs=5
NumRules=264
AndMethod='min'
OrMethod='max'
ImpMethod='min'
AggMethod='max'
DefuzzMethod='centroid'
```

[System] reflects the general framework of the FIS as a Mamdani-type system. It specifies the exact mathematical operators used for conjunction, disjunction, implication, aggregation, and the method of defuzzification.

```
[Input1]
Name='Man-Experience'
Range=[0 2500]
NumMFs=3
MF1='Basic':'trapmf',[-1132 -126.9 600 1250]
MF2='Senior':'trapmf',[600 1250 1750 2250]
MF3='Master':'trapmf',[1750 2250 2500 3625]

[Input2]
Name='Man-Composition'
Range=[0 3]
NumMFs=3
MF1='Tier-1':'trimf',[-1.2 0 1.2]
MF2='Tier-2':'trimf',[0.3 1.5 2.7]
MF3='Tier-3':'trimf',[1.8 3 4.2]
```

[Input1] and [Input2] are the first two of 15 KRIs belonging to the model. The naming convention specifies the associated RF (in this case, *Man*). Range reflects the domain of the universe of discourse. The number of constituent membership functions are specified, named, and parameterized. [Input1]'s are trapezoidal while [Input2]'s are triangular. The parameters represent the degree of truth at each vertex.

```
[Input3]
Name='Man-Endurance'
Range=[0 24]
NumMFs=4
MF1='Pumpkin':'trapmf',[-7.2 -0.8 5 8]
MF2='Fatigued':'trimf',[6 9 12]
MF3='Alert':'trapmf',[8 12 16 24]
MF4='Rested':'trimf',[16 24 31.99]

[Input4]
Name='Man-Currency'
Range=[0 90]
NumMFs=3
MF1='Recent':'trimf',[-36 0 36]
MF2='Currrent':'trimf',[0 30 60]
MF3='Uncurrent':'trapmf',[45 60 93.6 122]

[Input5]
Name='Machine-Readiness'
Range=[0 100]
NumMFs=3
MF1='NMC':'trapmf',[0 0 50 75]
MF2='PMC':'trimf',[50 75 100]
MF3='FMC':'trimf',[75 100 140]

[Input6]
Name='Machine-Performance'
Range=[0 20]
NumMFs=4
MF1='MTA<5'trimf',[-6.667 -2.22e-16 6.67]
MF2='MTA<10'trimf',[0 6.667 13.33]
MF3='MTA<15'trimf',[6.667 13.33 20]
MF4='MTA<20'trimf',[13.33 20 26.67]

[Input7]
Name='Medium-Illumination'
Range=[0 100]
NumMFs=4
MF1='Red':'trimf',[-33.33 0 20]
MF2='Amber':'trimf',[0 15 30]
MF3='Green':'trimf',[20 30 40]
MF4='Day':'trapmf',[30 50 103.3 130]

[Input8]
Name='Medium-Weather'
Range=[0 1500]
NumMFs=4
MF1='<500/1':'trimf',[-500 -7.105e-15 500]
MF2='>500/1':'trimf',[0 500 1000]
MF3='>700/2':'trimf',[500 750 1100]
MF4='>1000/3':'trapmf',[900 1100 1550 1950]
```

```
[Input9]
Name='Medium-Terrain'
Range=[0 15]
NumMFs=3
MF1='Improved':'trimf',[-6 0 6]
MF2='Adequate':'trimf',[1.48843484965305 7.48843484965305
13.488434849653]
MF3='Restricted':'trimf',[9 15 21]

[Input10]
Name='Management-Relationship'
Range=[0 3]
NumMFs=3
MF1='Assigned':'trimf',[-1.2 0 1.2]
MF2='Attached':'trimf',[0.3 1.5 2.7]
MF3='TACON':'trimf',[1.8 3 4.2]

[Input11]
Name='Management-Planning'
Range=[0 96]
NumMFs=4
MF1='Hasty':'trimf',[-32.1 -0.074 4]
MF2='Short':'trimf',[0 12 24]
MF3='Average':'trimf',[12 36 72]
MF4='Deliberate':'trapmf',[48 72 99.2 124.8]

[Input12]
Name='Management-Guidance'
Range=[0 3]
NumMFs=3
MF1='Specific':'trimf',[-1.2 0 1.2]
MF2='Implied':'trimf',[0.3 1.5 2.7]
MF3='Vague':'trimf',[1.8 3 4.2]

[Input13]
Name='Mission-Threat'
Range=[0 24]
NumMFs=3
MF1='Low':'trimf',[-9.6 0 6.4]
MF2='Moderate':'trimf',[4 11 18]
MF3='High':'trimf',[14 24 33.6]

[Input14]
Name='Mission-Complexity'
Range=[0 10]
NumMFs=4
MF1='Simple':'trimf',[-3.333 -1.11e-16 3.333]
MF2='Routine':'trimf',[0 3.333 6.667]
MF3='Irregular':'trimf',[3.333 6.667 10]
MF4='Elaborate':'trimf',[6.6592898997687 9.9922898997687
13.3222898997687]
```

```
[Input15]
Name='Mission-Type'
Range=[0 5]
NumMFs=5
MF1='BFC/Continuation':'trimf',[-1.25 0 1.25]
MF2='MTF/RL-Prog':'trimf',[0 1.25 2.5]
MF3='Air-Movement':'trimf',[1.25 2.5 3.75]
MF4='AASLT':'trimf',[2.5 3.75 5]
MF5='QRF/POI':'trimf',[3.75 5 6.25]

[Output1]
Name='Man'
Range=[0 10]
NumMFs=4
MF1='Low':'trimf',[-3.333 0 3.333]
MF2='Medium':'trimf',[0 3.333 6.667]
MF3='High':'trimf',[3.333 6.667 10]
MF4='Extreme':'trimf',[6.667 10 13.33]
```

[Output1] is the first of five Risk Factor linguistic variables belonging to the model. The naming convention, *Man*, indicates its association with the first four KRIs. Like those for the input variables, *Range* reflects the domain of the universe of discourse. The number of constituent membership functions (in this case, *NumMFs* = 4) are specified, named, and parameterized. The parameters represent the degree of truth at each of the triangular membership (‘*trimf*’) function’s three vertices.

```
[Output2]
Name='Machine'
Range=[0 10]
NumMFs=4
MF1='Low':'trimf',[-3.333 0 3.333]
MF2='Medium':'trimf',[0 3.333 6.667]
MF3='High':'trimf',[3.333 6.667 10]
MF4='Extreme':'trimf',[6.667 10 13.33]

[Output3]
Name='Medium'
Range=[0 10]
NumMFs=4
MF1='Low':'trimf',[-3.333 0 3.333]
MF2='Medium':'trimf',[0 3.333 6.667]
MF3='High':'trimf',[3.333 6.667 10]
MF4='Extreme':'trimf',[6.667 10 13.33]

[Output4]
Name='Management'
Range=[0 10]
NumMFs=4
MF1='Low':'trimf',[-3.333 0 3.333]
MF2='Medium':'trimf',[0 3.333 6.667]
MF3='High':'trimf',[3.333 6.667 10]
MF4='Extreme':'trimf',[6.667 10 13.33]

[Output5]
Name='Mission'
Range=[0 10]
NumMFs=4
MF1='Low':'trimf',[-3.333 0 3.333]
MF2='Medium':'trimf',[0 3.333 6.667]
MF3='High':'trimf',[3.333 6.667 10]
MF4='Extreme':'trimf',[6.667 10 13.33]
```

[Rules] consist of the 264 expert risk judgments that constitute the model's inferential logic. In this compact notation, the first series of 15 values represent the 15 KRIs as the proposition's antecedent. The next 5 values, comma delineated from the former, represent the model's 5 RFs. The cardinality of each indicates the specific subordinate membership function called by the proposition. The parenthetical value (1) shows the rule weight. The final value ':1' indicates the logical connective utilized (in this case, exclusively conjunctive).

```
[Rules]
1 1 1 1 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 1 1 2 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 1 1 3 0 0 0 0 0 0 0 0 0 0 0, 4 0 0 0 0 (1) : 1
1 1 2 1 0 0 0 0 0 0 0 0 0 0 0, 2 0 0 0 0 (1) : 1
1 1 2 2 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 1 2 3 0 0 0 0 0 0 0 0 0 0 0, 4 0 0 0 0 (1) : 1
1 1 3 1 0 0 0 0 0 0 0 0 0 0 0, 2 0 0 0 0 (1) : 1
1 1 3 2 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 1 3 3 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 1 4 1 0 0 0 0 0 0 0 0 0 0 0, 1 0 0 0 0 (1) : 1
1 1 4 2 0 0 0 0 0 0 0 0 0 0 0, 2 0 0 0 0 (1) : 1
1 1 4 3 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 2 1 1 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 2 1 2 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 2 1 3 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 2 2 1 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
1 2 2 2 0 0 0 0 0 0 0 0 0 0 0, 3 0 0 0 0 (1) : 1
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REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From – To)*
21-03-2019	Master's Thesis	September 2017 - March 2019

4. TITLE AND SUBTITLE

Fuzzy Inference Systems for Risk Appraisal in Military Operational Planning

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Nelson, Curtis B., Major, USA

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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WPAFB, OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER

AFIT-ENS-MS-19-M-141

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

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10. SPONSOR/MONITOR'S ACRONYM(S)

JWAC

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Distribution Statement A. Approved for Public Release; Distribution Unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Advances in computing and mathematical techniques have given rise to increasingly complex models employed in the management of risk across numerous disciplines. While current military doctrine embraces sound practices for identifying, communicating, and mitigating risk, the complex nature of modern operational environments prevents the enumeration of risk factors and consequences necessary to leverage anything beyond rudimentary risk models. Efforts to model military operational risk in quantitative terms are stymied by the interaction of incomplete, inadequate, and unreliable knowledge. Specifically, it is evident that joint and inter-Service literature on risk are inconsistent, ill-defined, and prescribe imprecise approaches to codifying risk. Notably, the near-ubiquitous use of risk matrices (along with other qualitative methods), are demonstrably problematic at best, and downright harmful at worst, due to misunderstanding and misapplication of their quantitative implications. The use of fuzzy set theory is proposed to overcome the pervasive ambiguity of risk modeling encountered by today's operational planners. Fuzzy logic is adept at addressing the problems caused by imperfect and imprecise knowledge, entangled causal relationships, and the linguistic input of expert opinion. To this end, a fuzzy inference system is constructed for the purpose of risk appraisal in military operational planning.

15. SUBJECT TERMS

operational risk, risk assessment, fuzzy logic, fuzzy inference systems

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Dr. Richard F. Deckro AFIT/ENS
U	U	U	UU	123	**19b. TELEPHONE NUMBER** *(include area code)* 937-255-6565 x4325 richard.deckro@afit.edu

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18